16.21 - Techniques of structural analysis and design
Homework assignment # 2
Handed out: 2/18/05
Due: 2/25/05

February 17, 2005

1. Determine whether the following stress fields are possible in a structural member free of body forces:

   (a) (not for grade)

$$\sigma_{11} = -3x_1 + 6x_2 \tag{1}$$
$$\sigma_{12} = 4x_1 + 3x_2 \tag{2}$$
$$\sigma_{22} = 5x_1 + 4x_2 \tag{3}$$

   (b)

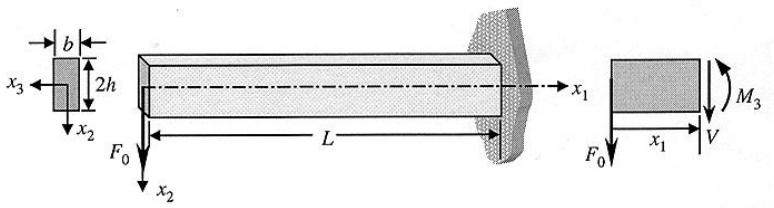
$$\sigma_{11} = c_1 x_1 + c_2 x_2^2 + c_3 x_1 x_2 + c_4 x_1 \tag{4}$$
$$\sigma_{12} = -\frac{c_3}{2} x_2^2 - c_1 x_2^2 - c_4 x_2 \tag{5}$$
$$\sigma_{22} = c_4 x_1 + c_1 x_2 \tag{6}$$

(c)

$$\sigma_{11} = 2x_1^2 - 2x_1x_2 + 6x_3 \tag{7}$$
$$\sigma_{12} = -x_1x_2 + x_2^2 \tag{8}$$
$$\sigma_{13} = x_1x_3 \tag{9}$$
$$\sigma_{22} = -3x_2^2 \tag{10}$$
$$\sigma_{23} = -5x_2x_3 \tag{11}$$
$$\sigma_{33} = 2(x_1 - 2x_2)x_3 \tag{12}$$

2. Given the following state of stress, determine the body forces for which the stress field describes a state of equilibrium:

$$\sigma_{11} = -4x_1^2 \tag{13}$$
$$\sigma_{12} = x_1^3 + x_2^2 + 2x_1x_2 \tag{14}$$
$$\sigma_{13} = -4x_1 + 2x_2^2 - 7x_3 \tag{15}$$
$$\sigma_{22} = 3x_2^2 - 2x_3^2 \tag{16}$$
$$\sigma_{23} = 4x_1x_2x_3 \tag{17}$$
$$\sigma_{33} = (2x_1 - 3x_2)x_3 \tag{18}$$



3. For the cantilever beam loaded with a point load at the free end (see figure), the bending moment M_3 about the x_3-axis is given by $M_3 = -F_0x_1$. The bending stress σ_{11} is given by:

$$\sigma_{11} = \frac{M_3x_2}{I_3}$$

where I_3 is the moment of inertia of the cross section about the x_3-axis. Use the two-dimensional equilibrium equations in differential form to determine the stress fields: σ_{22} and σ_{12}.

4. For the state of stress of question 3, determine the stress vector and its normal and shear components at the point $(L, h, 0)$ on the plane of normal:

   (a) $(1, 0, 0)$

   (b) $\frac{1}{\sqrt{3}}(1, 1, -1)$

   Determine the principal stresses and principal directions of stress at this point.