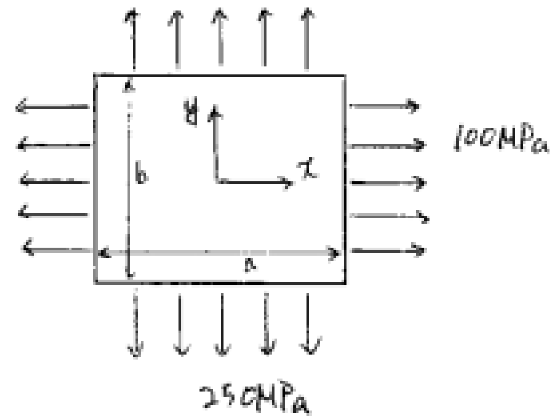
Practice Problems

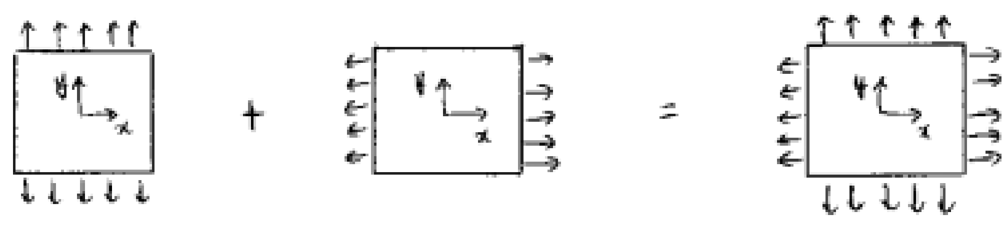
5. Pressurized vessel under plane stress with 100MPa in one direction and 250MPa perpendicular to this direction.



$E = 70GPa$

$\nu = 0.3$

i) To construct a solution for this configuration, we need to refer to the Airy stress function sheet. From this sheet, we can see that the biaxial tension state of the current problem can be described by the superposition of uniaxial tension problems.



Stress function: $\quad C_{20}x^2 \quad + \quad C_{02}y^2 \quad = \quad C_{20}x^2 + C_{02}y^2$

* Note: The stress functions can be superposed because the problem is linear.

Thus, the stress function for this problem is

$$\phi = C_{20} x^2 + C_{02} y^2 \qquad \text{--------- } (1)$$

From the stress function in equation (1), we can find the stresses.

$$\sigma_{xx} = 2C_{02} \qquad \text{--------- } (2)$$

$$\sigma_{yy} = 2C_{20} \qquad \text{--------- } (3)$$

$$\sigma_{xy} = 0 \qquad \text{--------- } (4)$$

Using the given stress-state and from equation (2) and (3), we can find the constants C_{02} and C_{20}.

$$\sigma_{xx} = 2C_{02} = 100\,MPa \qquad \text{--------- } (5)$$

$$\Rightarrow \quad C_{02} = 50\,MPa$$

$$\sigma_{yy} = 2C_{20} = 250\,MPa \qquad \text{--------- } (6)$$

$$\Rightarrow \quad C_{20} = 125\,MPa$$

Therefore, the stress function is

$$\boxed{\phi = (50MPa)y^2 + (125MPa)x^2}$$

b) The stress-strain relations can be expressed as follows (for isotropic & plane stress)

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) \quad \text{—— ⑦}$$

$$\varepsilon_{yy} = \frac{1}{E}(-\nu\sigma_{xx} + \sigma_{yy}) \quad \text{—— ⑧}$$

$$\varepsilon_{zz} = \frac{1}{E}(-\nu\sigma_{xx} - \nu\sigma_{yy}) \quad \text{—— ⑨}$$

$$\varepsilon_{xz} = 0 \quad \text{—— ⑩}$$

$$\varepsilon_{yz} = 0 \quad \text{—— ⑪}$$

$$\varepsilon_{xy} = \frac{1}{G}\sigma_{xy} \quad \text{—— ⑫}$$

Plugging in the values for σ_{xx}, σ_{yy}, σ_{xy}, and E and ν, we get

$$\varepsilon_{xx} = \frac{1}{70GPa}(100MPa - 0.3 \times 250MPa)$$

$$\Rightarrow \boxed{\varepsilon_{xx} = 360 \times 10^{-6}}$$

$$\varepsilon_{yy} = \frac{1}{70GPa}(-0.3 \times 100MPa + 250MPa)$$

$$\Rightarrow \boxed{\varepsilon_{yy} = 3100 \times 10^{-6}}$$

$$\varepsilon_{zz} = \frac{1}{70GPa}\left(-0.3\times 100MPa - 0.3\times 250MPa\right)$$

$$\Rightarrow \boxed{\varepsilon_{zz} = -1500\times 10^{-6}}$$

$$\varepsilon_{xy} = \frac{1}{2G}\,0$$

$$\Rightarrow \boxed{\varepsilon_{xy} = 0}$$

Summarizing, the strain state is

$$\boxed{\begin{aligned} \varepsilon_{xx} &= 360\times 10^{-6} \\ \varepsilon_{yy} &= 3100\times 10^{-6} \\ \varepsilon_{zz} &= -1500\times 10^{-6} \\ \varepsilon_{xy} &= 0 \end{aligned}}$$

c) The strain-displacement relations are as follows.

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \quad\text{———} \quad (13)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \quad\text{———} \quad (14)$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \quad\text{———} \quad (15)$$

$$\varepsilon_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \qquad (16)$$

$$\varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \qquad (17)$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \qquad (18)$$

Plugging in the strains and rearranging equations (13) through (18), we get

$$\frac{\partial u}{\partial x} = 360 \times 10^{-6} \qquad (19)$$

$$\frac{\partial v}{\partial y} = 3100 \times 10^{-6} \qquad (20)$$

$$\frac{\partial w}{\partial z} = -1500 \times 10^{-6} \qquad (21)$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \qquad (22)$$

$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \qquad (23)$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \qquad (24)$$

A solution that satisfies equations (19) through (24) is

$$\boxed{\begin{aligned} u &= 360 \times 10^{-6} x \\ v &= 3100 \times 10^{-6} y \end{aligned}}$$

$$\boxed{w = -1500 \times 10^{-6} z}$$

* Note: An arbitrary constant can be added to the displacements. The arbitrary constant represents rigid body motion.