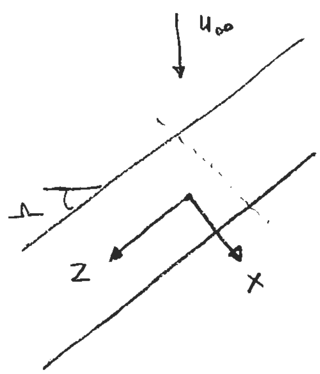
3D Boundary Layers

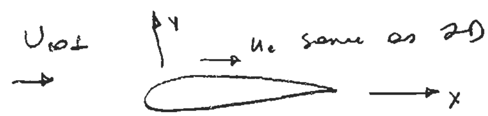
A) Infinite Swept Wing

~~B) Sweep and Taper~~

B) Strip Theories (Quasi-3D)

A) Infinite Swept Wing.

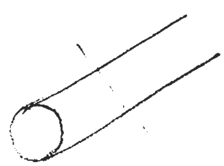




$$U_{\infty\perp} = U_\infty \cos\Lambda$$

$$U_{\infty\parallel} = U_\infty \sin\Lambda$$

$$W_e(x,z) = U_{\infty\parallel}$$



$$\frac{D}{\text{length}} = \frac{1}{2}\rho U_{\infty\perp}^2 C_D \sim \cos^2\Lambda$$

TSL Equations

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \cancel{\frac{\partial \rho w}{\partial z}} = 0$$

$$\frac{\partial}{\partial z} = 0 \quad \text{by geometry}$$

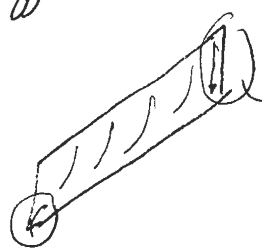
$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \cancel{\rho w\frac{\partial u}{\partial z}} = \rho u_e\frac{\partial u_e}{\partial x} + \cancel{\rho w_e\frac{\partial u_e}{\partial z}} + \frac{\partial \tau_x}{\partial y}$$

$$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = \rho u_e \frac{\partial u_e}{\partial z} + \rho_e w_e \frac{\partial w_e}{\partial z} + \frac{\partial \tau_z}{\partial y}$$

$\Rightarrow$ x – z momentum uncoupled

Solve using F-S along x with const crossflow in z

Strong 3D effects



Thicker BL prone to sep. – BL gets squeezed (very neg $\frac{\partial w}{\partial z}$)

- No coupling in 2D
- Turbulent coupling via Reynold's stresses (weak)

B) Const. Cross flow Approx

Curvilinear transformation from $(x, y, z) \to (x', y', z')$

Intrinsic coordinates $x' = \int \frac{u_e}{|\vec{q}_e|} dx + \int \frac{w_e}{|\vec{q}_e|} dz$

$$z' = \int \frac{u_e}{|\vec{q}|} dz - \int \frac{w_e}{|\vec{q}|} dx$$

x' aligned with $\vec{q}_e$

- Assume $w_e = 0$, $\frac{\partial}{\partial z'} = 0$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

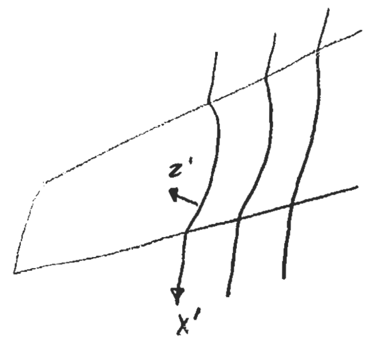
$$\rho \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - uwK_2 + u^2 K_1 \right] = \rho_e u_e \frac{du_e}{\partial x} + \frac{\partial \tau_{x'}}{\partial y}$$

$$\rho\left[u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+u^2K_2-uwK_1\right]=\rho_e u_e^2K_2+\frac{\partial \tau_{z'}}{\partial y}$$

- K_1, K_2 are coordinate line curvatures

⇒ We have 2D problem along each z' = const line

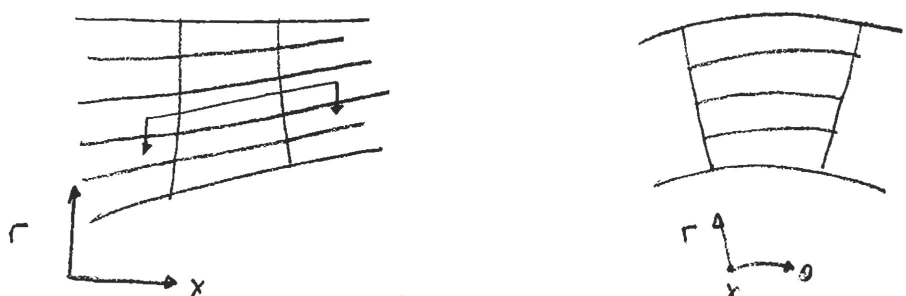
Extra unknown : crossflow W (not small)
" " : z-mom

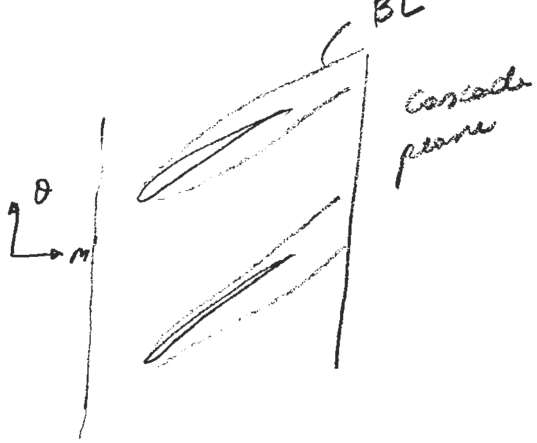


2-D BL strips

Turbomach Q3D

Flow along streamsurfaces (axisymmetric)



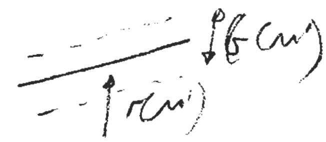


$$m'=\int\frac{dm}{r}=\int\frac{\sqrt{dx^2+dr^2}}{r}$$

meridional coordinate

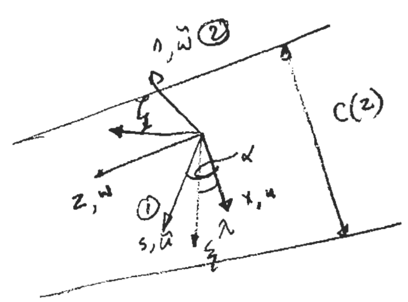
Solve quasi3D cascade problem along streamsurface

Quasi-3D IBL eqns:



$r(m')$, $b(m')$ — streamtube thickness

3D Sweep & Taper



ζ

ζ

ζ

Local streamwise coordinates (s,η) , $(\hat{u},\hat{w})$

$$u = \hat{u}\cos\alpha - \hat{w}\sin\alpha \qquad \cos\alpha = \frac{u_e}{q_e} \qquad \hat{u}_e = q_e = \sqrt{u_e^2 + w_e^2}$$

$$w = \hat{u}\sin\alpha + \hat{w}\cos\alpha \qquad \sin\alpha = w_e/q_e \qquad \hat{w}_e = 0$$

Transformed integral thickness

Example

$$\rho_e q_e \delta_x^* = \int (\rho_e u_e - \rho u)\,dy = \rho u_e \delta_1^* - \rho w_e \delta_2^*$$

substitute transformed velocity

Similarly, other thickness .... (see handout)

Note profiles defined in local streamwise coordinates / direction

The integral thickness in (x, y) system can be calculated

Convenient integration coordinates are ξ, ζ

$$\xi = x \cos\lambda + z \sin\lambda$$
$$\zeta = -x \sin\lambda + z \cos\lambda$$

Transform derivatives

$$\frac{\partial}{\partial x}(\ ) =$$

$$\frac{\partial}{\partial z}$$

Approximation in the case of infinite, yawed, tapered