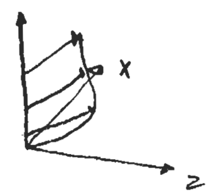
Lecture 33 — Dec 3, 03.

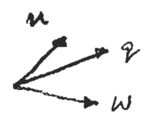
3D Boundary Layers.

A> 3D Integral BL Equations

B> Implications for 3D drag.

Ref: Mughal, B Ph.D 1998.





$q^2 = u^2 + w^2$

A> 3D Int BL Eqns

$(u - u_e)\cdot$ cont $+$ x-mom

$$\frac{\partial}{\partial x}\left[\rho u (u-u_e)\right] + \frac{\partial}{\partial y}\left[\rho v (u-u_e)\right] + \frac{\partial}{\partial z}\left[\rho w (u-u_e)\right]$$

$$+ (\rho_e u_e - \rho u)\frac{\partial u_e}{\partial x} + (\rho_e w_e - \rho w)\frac{\partial w_e}{\partial x} - \frac{\partial \tau_{xy}}{\partial y} = 0$$

$= \frac{\partial u_e}{\partial z}$ in pot flow

Integrating in y gives

$$\frac{\partial}{\partial x}\left[\rho_e q_e^2 \theta_{xx}\right] + \frac{\partial}{\partial z}\left[\rho_e q_e^2 \theta_{xz}\right] + \rho_e q_e \delta_x^* \frac{\partial u_e}{\partial x} + \rho_e q_e \delta_z^* \frac{\partial u_e}{\partial z} = \tau_{xy_w}$$

Similary for z-mom

$$\frac{\partial}{\partial x}\left(\rho_e q_e^2 \theta_{zx}\right) + \frac{\partial}{\partial z}\left[\rho_e q_e^2 \theta_{zz}\right] + \rho_e q_e \delta_x^* \frac{\partial w_e}{\partial x} + \rho_e q_e \delta_z^* \frac{\partial w_e}{\partial z} = \tau_{zw}$$

Energy eqn

$(q^2 - q_e^2)$ cont. $+ 2u\,[\text{x-mom}] + 2w\,[\text{z-mom}]$

$$\Rightarrow \frac{\partial}{\partial x}\left[\rho_e q_e^3 \theta_x^*\right] + \frac{\partial}{\partial z}\left[\rho_e q_e^3 \theta_z^*\right] + \rho_e q_e \delta_x^{**} \frac{\partial q_e^2}{\partial x} + \rho_e q_e \delta_z^{**} \frac{\partial q_e^2}{\partial z} - 2D = 0$$

Definitions

$$q_e \delta^*_x = u_e \int \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy \qquad q_e \delta^*_z = w_e \int \left(1 - \frac{\rho w}{\rho_e w_e}\right) dy$$

$$q_e^2 \theta_{xx} = u_e^2 \int \left(1 - u/u_e\right) \frac{\rho u}{\rho_e u_e}\, dy \qquad q_e^2 \theta_{xz} = u_e w_e \int \left(1 - u/u_e\right) \frac{\rho w}{\rho_e w_e}\, dy$$

$$q_e^2 \theta_{zz} = w_e^2 \int \left(1 - \frac{w}{w_e}\right) \frac{\rho w}{\rho_e w_e}\, dy \qquad q_e^2 \theta_{zx} = u_e w_e \int \left(1 - w/w_e\right) \frac{\rho u}{\rho_e u_e}\, dy$$

don't commute.

$$q_e \theta^* = u_e \int \left(1 - q^2/q_e^2\right) \frac{\rho u}{\rho_e u_e}\, dy \qquad q_e \theta_z^* = \int \left(1 - q^2/q_e^2\right) \frac{\rho w}{\rho_e w_e}\, dy\, w_e$$

$$D = \int \left(\tau_x \frac{\partial u}{\partial y} + \tau_z \frac{\partial w}{\partial y}\right) dy$$

Identity:

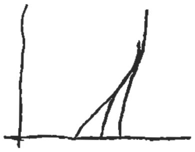
$$q_e \left(\theta_{xz} - \theta_{zx}\right) = w_e \delta^*_x - u_e \delta^*_z$$

Compare with 2D

- 2-D PDE's in (x, y)
- 1-D Integral ODE in x
- 2 eqn
  $\frac{d\theta}{dx} = \ldots$
  $\frac{d\theta^*}{dx} = \ldots$
- 2 unknowns $\theta(x)$, $\delta^*(x)$
- 3 correlations: θ^* or H^*
  C_f
  C_D

- 3D PDEs in (x, y, z)
- 2-D integ PDE in (x, z)
- 3 eqn: x-mom $\frac{\partial \theta_{xx}}{\partial x} + \ldots$
  z-mom $\frac{\partial \theta_{zz}}{\partial z} + \ldots$
  energy $\frac{\partial \theta^*_x}{\partial x} + \frac{\partial \theta^*_z}{\partial z} + \ldots$
- 3 unk: $\theta_{xx}(x, z)$
  $\delta^*_x(x, z)$
  $\delta^*_y(x, z)$
- 7 correlations for: θ_{xz}, θ_{zz}, θ^*_x, θ^*_z, C_{fx}, C_{fz}, C_D

$\theta^* \equiv H^* \cdot \theta$ from 1 parameter outer profile U



Ex: Cole's

$H \rightarrow$ profile $\rightarrow H^*$

- Need profiles for U & W

$U_e \equiv (u_e/q_e) = 1$ – defined along outer streamline

$W_e \equiv \dfrac{w_e}{q_e} = 0$

- cross flow shape parameter

$\beta = \delta_z^* / \theta_{xx}$

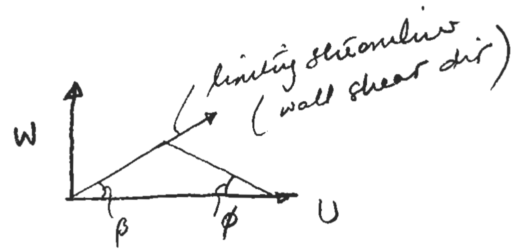
Possible profiles

- Mager

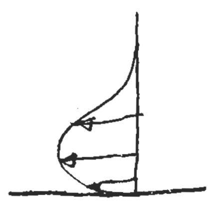
$$W = U\left(1 - y/\delta\right)^2 \tan\beta$$

- Johnston

$$W = U \tan\beta$$
$$W = (1-U)\tan\phi$$



ϕ is some fraction of β.
– scales magnitude of crossflow.



So $\quad U = U\left(y\,;\ \delta_x^*/\theta_{xx}\right)$

$\quad W = W\left(y\,;\ \delta_x^*/\theta_{xx},\ \delta_z^*/\theta_{xx}\right)$

Use U, W to get all other thicknesses in terms of $\underbrace{\theta_{xx},\ \delta_x^*,\ \delta_z^*}_{\text{unknowns}}$

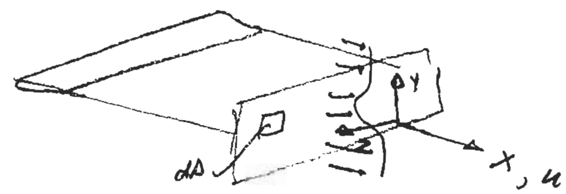
C_{f_x} from 2D.
$C_{f_z} = C_{f_x}\tan\beta$
C_D from $U, W, \tau/\tau_t$

Closed system for $\theta_{xx}, \delta^*_x, \delta^*_z$ – 3 PDEs

3D method far more complicated than 2D !

Other Refs: McLean & Randall NASA CR 3123
P. D. Smith ARC R&M 3739

B) 3D Drag.



$$D = \int (U_\infty - U)\, d\dot{m}$$

$$= \iint (U_\infty - u)\, \rho u\, dy\, dz$$

$$= \int_{\text{span}} \rho u_e^2\, \theta_{xx}\, dz$$

$$\frac{\partial}{\partial x}\left(\rho_e q_e^2 \theta_{xx}\right) = \tau_{xy_w} - \frac{\partial}{\partial z}\left[\rho_e q_e^2 \theta_{xz}\right] - \rho_e q_e \delta^*_x \frac{\partial u_e}{\partial x} - \rho_e q_e \delta^*_z \frac{\partial u_e}{\partial z}$$

Formally integrate

momentum redistribution

$$\iint \rho_e q_e^2 \theta_{xx}\, dx\, dz = \iint \tau_{xy_w}\, dx\, dz - \iint \frac{\partial}{\partial z}\left[\rho_e q_e^2 \theta_{xz}\right] dx\, dz$$
$$- \iint \left(\rho_e q_e \delta^*_x \frac{\partial u_e}{\partial x}\right) dx\, dz$$
$$- \iint \left(\rho_e q\, \delta^*_z \frac{\partial u_e}{\partial z}\right) dx\, dz.$$

$$\int \rho_e q_e^2 \theta_{xx}\Big|_{\text{wake}} dz = \underbrace{\iint \tau_{x_w}\, dx\, dz}_{\text{friction}} - \underbrace{\int\left(\int \rho_e q_e \delta^*_x \frac{\partial u_e}{\partial x}\, dx\right) dz}_{\text{usual 2D pressure drag.}}$$

$$- \iint \Big(\qquad \qquad \Big) \text{ new 3D pressure}$$

$$\int dx \iint \overbrace{(\rho_e w_e - \rho w)\, dy}^{\rho_e q_e \delta_z^*} \frac{\partial u_e}{\partial z}\, dz .$$

- Drag generated anytime there is crossflow in the presence of transverse pressure gradient – swept wing.