Lecture 23.

A) Transition Mechanisms / Phenomena

, Transition Prediction

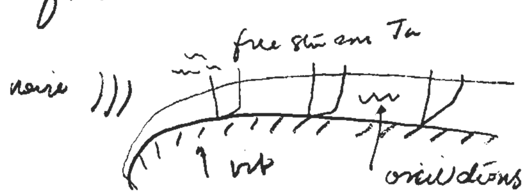
Reading : Sch
Wh
Handouts

A) Phenomena , Transition Mechanisms

- Transition prediction is difficult because of large number of uncertainties. There is no exact theoretical method that can be used to predict transition. Empirical methods that work.

  In general transition is effected by
  - free stream turb
  - pressure gradient
  - surf. curvature
  - roughness , temp
  - noise
  - vibration
  - compressibility etc.

Process for "free" or "natural" transition



Ambient Disturbances
- noise
- vibration
- free stream turbulence

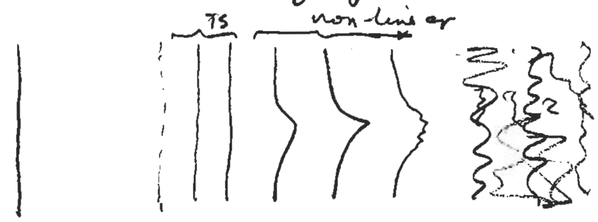
- These disturbances become seed to TS waves via receptivity (non-linear) process

TS wave λ in mm

Disturbance λ in meters

1. Initial wave amplitude via receptivity
2. Exponential growth for certain combinations of Re, ω_r, H
3. Non-linear breakdown (2nd order terms become significant
4. Fully Turbulent flow

See White (pg 376)



- In addition, separation can induce transition (pseudo-natural)

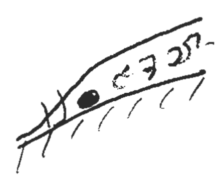
.. Bypass Transition

Initial forcing already at non-linear level (no weakly affected by pressure gradient

Typical in high turb intensity environment like turbomachinery > 1st stage

... "forced" transition – via turbulators, trip strips, roughness, etc.

– direct breakdown of laminar flow



B> Transition Prediction

A> Simple correlation

B> Amplification Method

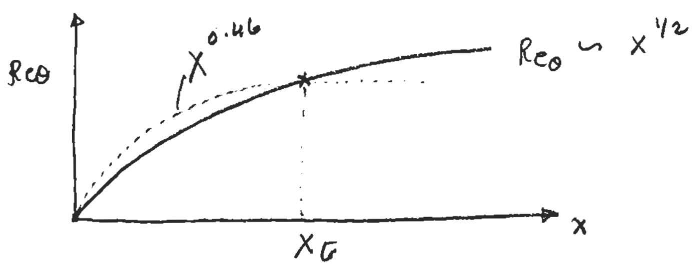
C> Bypass Method

① Simple Correlation (one step method)

Michel's criterion: transition occurs when
(1952)

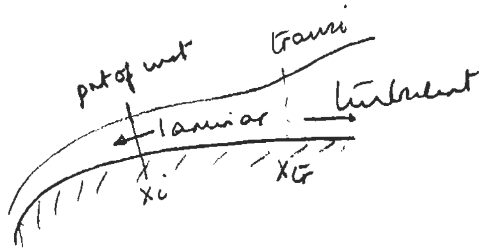
$$Re_\theta \geq Re_{\theta_{trans}} = 1.174 \left(1 + \frac{22400}{Re_x} \right) Re_x^{0.46}$$

where $Re_x = \frac{U_e(X) \cdot X}{\nu}$



Note: 1) X is arbitrary
2) works for Blasius like flow } limited application

2 step method (Granville)
1953



- calculate X_i until $Re_{\theta crit}$ (H) using Thwaites method
- $$\lambda_m = \frac{1}{X_{tr} - X_i} \int_{X_i}^{X_{tr}} \lambda(x)\, dx \qquad \text{(mean)}$$

$$\lambda = \frac{\theta^2}{\nu} \frac{dU}{dX}$$

- Transition when

fit to empirical data $\longrightarrow$ $Re_{\theta_x} \geq Re_{\theta_{tr}} = Re_\theta(x_i) + 450 + 400\, e^{60 \lambda_m}$

$$\frac{d\,Re_\theta}{d\xi} = \frac{d}{d\xi}\left(\frac{\rho u_e \theta}{\mu_0}\right) = \frac{\rho}{\mu}\frac{d}{d\xi}(u_e\theta)$$

$$\theta = \theta_1 \cdot \xi^{(1-m)/2}$$

$$= \theta_1 \cdot \xi^{(1-\beta u)2}$$

$$u_e = \xi^{\beta u}$$

$$u_e\theta = C \cdot \xi^{\frac{1-\beta u}{2}} \cdot \xi^{\beta u}$$

$$= C\ \xi^{\frac{1+\beta u}{2}}$$

$\beta u - 1 - 1 - \beta u$

$$\frac{d\,u_e\theta}{d\xi} = C\left(\frac{1+\beta u}{2}\right) \cdot \xi^{\frac{1+\beta u - 2}{2}} \cdot \beta\frac{u-1}{2}$$

$$= C\left(\frac{1+\beta u}{2}\right)\xi^{\beta\frac{u-1}{2}}$$

$$= u_e\theta \cdot \frac{\xi^{\beta\frac{u-1}{2}}}{\xi^{\frac{1+\beta u}{2}}} \cdot \left(\frac{1+\beta u}{2}\right)$$

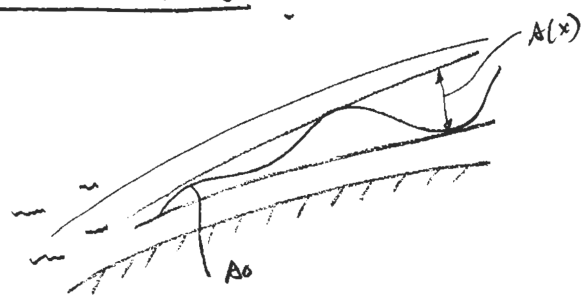
$$\frac{d\,Re_\theta}{d\xi} = \frac{\rho u_e \theta}{\mu\,\xi}(1+\beta u)\frac{1}{2}$$

④

$\lambda = -0.1 \rightarrow$ adverse pressure gradient, last term negligible

so transition moves close to x_i.

B) <u>Amplification Methods</u>



- Assume background disturbance level
- Assume each frequency grows independantly
- <u>Key</u>: linear growth region dominates process (unlike bypass, forced)
natural

Transition occurs when $A(x; \omega_r)$ for any frequency crosses a threshold value, where

$$A = |\hat{u}'| \text{ or } |\hat{v}|$$

Define threshold in terms of: $\frac{A(x)}{A_0} = e^n$

where $n = 9$ typically ($e^9 \approx 8100$)

Need to find $A(x)$ for given ω_r. Our assumed perturbation is

$$\hat{v} = \check{v}(y) e^{i(\alpha_r x - \omega_r t)} e^{-\alpha_i x}$$

$$\therefore \quad A = |\hat{v}| = |\check{v}(y)| \, e^{-\alpha_i x}$$

$$\ln A = \ln |\check{v}| - \alpha_i x$$

$$\frac{d \ln A}{dx} = -\alpha_i$$

We know $\quad \alpha_i \theta = \tilde{\alpha}_i^*(Re_\theta, H, \vec{\omega}^*) \qquad \omega^* = \frac{\omega_r \theta}{u_e}$ (use u_e, θ as ref scales)

soln of O-S eqn

$$\frac{A(x)}{A_0} = e^{n}$$

$$\ln \frac{A}{A_0} = n \quad \Rightarrow \quad \frac{d}{dx} \ln A = \frac{dn}{dx}$$

$$\ln A = \ln A_0 + \int_{x_0}^{x} \frac{d}{dx}(\ln A)\, dx$$

$$A = |\tilde{V}|\, e^{-\alpha_i x}$$

$$\ln A = \ln|\tilde{V}| \mp \alpha_i x$$

$$\frac{d \ln A}{dx} = -\alpha_i = \frac{dn}{dx}$$

$$\ln A = \ln A_0 - \int_{x_0}^{x} \alpha_i\, dx$$

$$\ln(A/A_0) = n = -\int_{x_0}^{x} \alpha_i\, dx$$

Since

$$\frac{A(x)}{A_0} = e^n \Rightarrow \ln\frac{A}{A_0} = n(x;\omega_r)$$

$$\therefore \frac{dn}{dx}(x;\omega_r) = -\alpha_i(Re_\theta, H, \omega_r)$$

O.S eqn has form

$$\alpha^*_i = \alpha^*_i(Re_\theta, H, \omega^*_r)$$

$$\Rightarrow n(x;\omega) = \int_{x_0}^{x} -\frac{\alpha^*_i(Re_\theta, \omega^*_r, H)}{\theta(x)}\,dx$$

Solution of O.S eqn

$$\ln A(x;\omega_r) = \ln A_0(\omega_r) + \int_{x_0(\omega_r)}^{x} \frac{d}{d\xi}(\ln A)\,d\xi$$

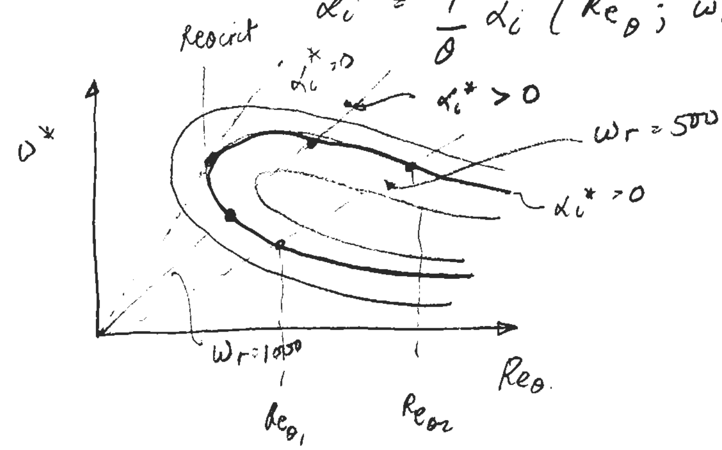
$$\therefore \ln\frac{A}{A_0} \equiv n = -\int_{x_0}^{x} \alpha_i\,d\xi$$

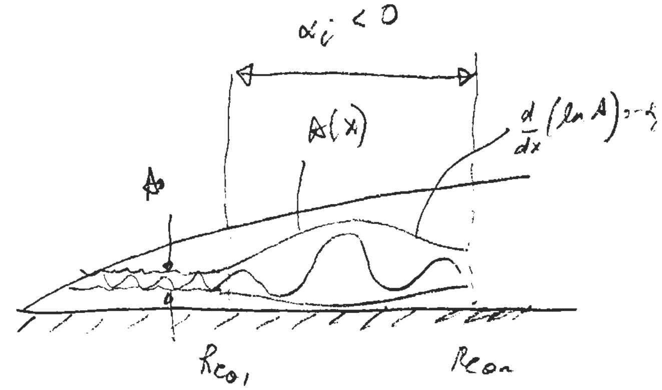
$$= \int_{x_0}^{x} -\frac{\alpha^*_i}{\theta}(Re_\theta, H, \omega^*_r)\,d\xi$$

x_0 location where $\alpha_i = 0$

Ex Similar flow (H fixed) (Blasius)

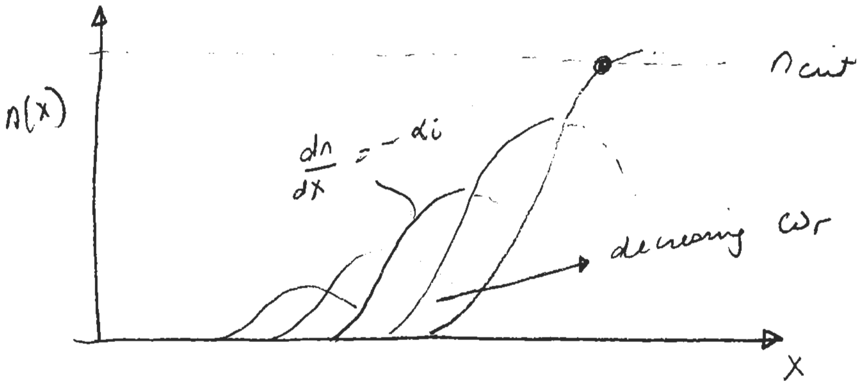
$$\alpha_i = \frac{1}{\theta}\bar{\alpha}^*_i(Re_\theta; \omega^*_r)$$





Look at 1 frequency ω_r (500 rad/s)

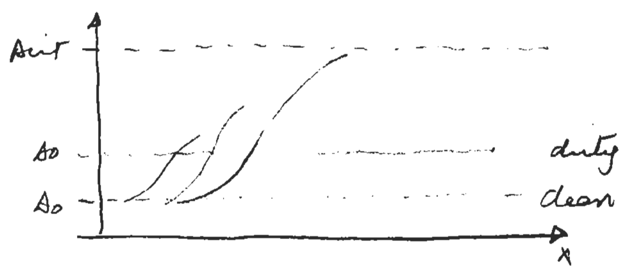
$$\omega^*(x) = \frac{\omega_r\,\theta(x)}{U_e(x)}, \quad Re_\theta(x) = \frac{U_e(x)\,\theta(x)}{\nu}$$



decreasing ω_r cuts through larger instability region

- In general, lower frequencies
  - go unstable farther downstream
  - grow more over longer distance

- Some frequency (•) will reach n_{crit} first triggering transition
- A_0 depends on ambient disturbance level. If A_{crit} is fixed then n_{crit} depends on disturbance level



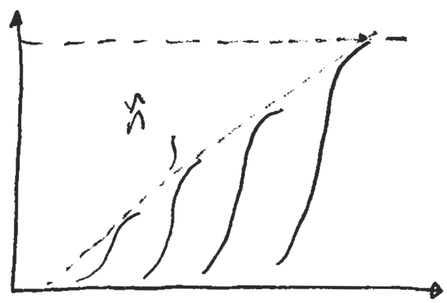
	N_{crit}
Sailplane	14 - 18
Powered airplane	12
Clean wind tunnel	9
Dirty tunnel	4-8
Jet engine	< 1

← n_{crit} 1st expt. determined

→ at end. pg 8

Referring to handout: the F-S velocity profiles coupled with O-S equation were solved, substituted into $n(x;\omega_r)$ and integrated. This is OK for similar flows. Non-similar flows on pg 3-4 airfoil case.

"e^n" is laborious and expensive. Simplification is to use the "envelop method"



$$\ln(A/A_0) \equiv \hat{n} = \frac{d\hat{n}}{dRe_\theta}(H)\left\{Re_\theta - Re_{\theta_0}(H)\right\}.$$

$$\frac{d\hat{n}}{dRe_\theta} = f_1(H) \qquad \text{Eqn } 6.42$$

$$Re_{\theta_0} = f_2(H) \qquad \text{Eqn } 6.43.$$

$$\frac{d\hat{n}}{dX} = \frac{d\hat{n}}{dRe_\theta}\cdot\frac{dRe_\theta}{dX}$$

For similar flows (F-S)

$$\frac{d}{dx}\left(\frac{\rho u_e \theta}{\mu_e}\right) = \frac{\rho}{\mu_e}\frac{d}{dx}(u_e\theta)$$

$$u_e \sim C X^M \qquad M = \beta_H$$

$$\theta \sim \theta_1 X^{(1-M)/2}$$

$$u_e\theta \sim X^{\frac{1+M}{2}} \qquad \left(C X^{\frac{1+M}{2}}\right)$$

$$\frac{d}{dx}u_e\theta = C\left(\frac{1+M}{2}\right)\cdot X^{\frac{M-1}{2}}$$

$$= \frac{u_e\theta}{X^{\frac{1+M}{2}}}\cdot X^{\frac{M-1}{2}} \quad \Rightarrow \quad \frac{dRe_\theta}{dX} = \frac{\rho u_e \theta}{\mu X}(1+\beta_H)\frac{1}{2}.$$

Additional equations (6.45) and (6.46) gives

$$\frac{d\tilde{n}}{dx}(H,\theta) = \frac{d\tilde{n}}{dRe_\theta}(H) \cdot \frac{m(H)+1}{2} \cdot \frac{l(H)}{\theta}$$

we can integrate

$$\tilde{n}(x) = \int_{X_0}^{X} \frac{d\tilde{n}}{dX}\, dX$$

until $\tilde{n} = 9$, which indicates onset of transition

Table of n_{crit} values.

Very dirty flows experience bypass transition ($Tu > 1$-2%)

Table

c, Bypass Method

Bypass transition occurs when background noise (disturbance) results in non-linear levels of u', v', w', p'. Free stream turbulence level $\gtrsim 1\%$

$$Tu(x) = \frac{\sqrt{\overline{u'^2}}}{U_e(x)} \times 100 \qquad u' - \text{RMS value}$$

Abu - Ghannam - Shaw Criterion: (Journal of Mech Eng Sci, May 1980)

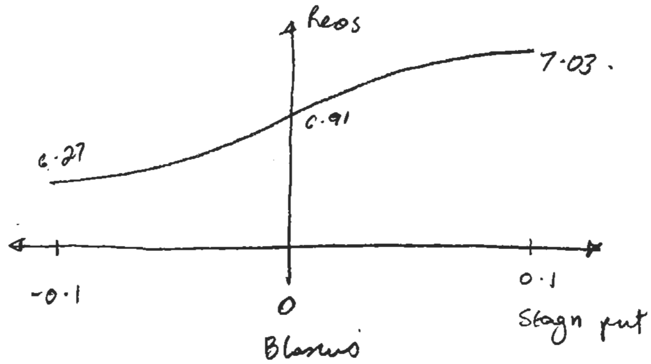
Transition occurs when: Thwaites parameter

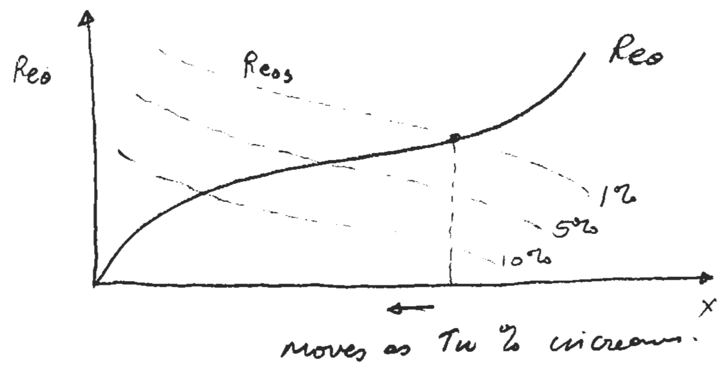
$$Re_\theta \geq Re_{\theta s}(\lambda(x), Tu(x))$$

$$\text{or } Re_{\theta s}(H(x), Tu(x))$$

$$Re_{\theta_s} = 163 + e^{[F(\lambda)(1 - Tu/6.91)]} \quad \leftarrow \text{empirical}$$

where $F(\lambda) = \begin{cases} 6.91 + 12.75\lambda + 63.64\lambda^2 & \lambda < 0 \\ 6.91 + 2.48\lambda - 12.27\lambda^2 & \lambda > 0 \end{cases}$

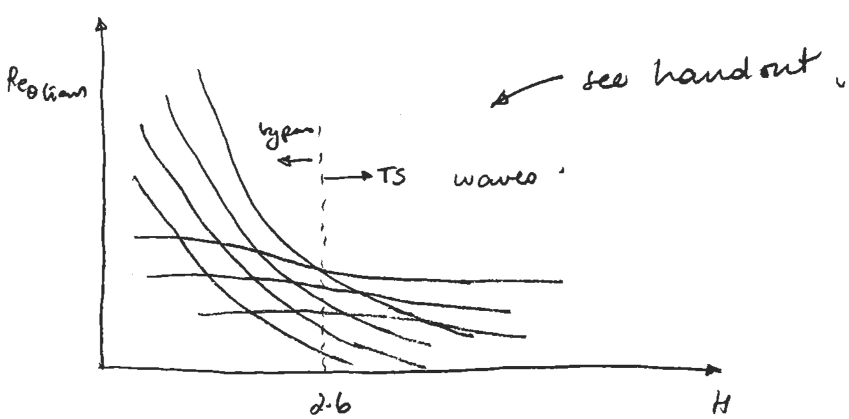




The AGS criterion is much less affected by pressure gradient than TS wave mechanism. (transition on turbine blades with accelerating flow.

Combine e^n method with bypass method, AGS criterion for range of Re_θ, Tu (or N_{crit}), H (similar flows)

Condition for small Tu : $n_{crit} = -8.43 - 2.4 \ln(Tu/100)$
(Mack) Make $N_{crit} = N_{crit}(Tu)$ valid for $Tu < 3\%$



Bypass mechanism more likely to cause transition in favorable pressure gradient $H < 2.6$

TS wave " " " adverse " gradient $(H > 2.6)$