Stability and Transition

A) Solution Technique

B) Promotion / Suppression

C) Transition

__Reading__: Sch – 434 – 473.
C & B – 290 – 301

A) __Soln Technique:__ Recall O-S eqn is 4th order complex ODE in y. We would like to solve for $v(y)$, α or ω for temporal or spatial problem given mean flow: Re, $U(y)$.

Rewrite O-S as a system of 1st order ODEs

$$(\alpha U - \omega)(v'' - \alpha^2 v) - \alpha U'' v$$
$$+ \frac{i}{Re}\left[v'''' - 2\alpha^2 v'' + \alpha^4 v\right] = 0$$

(φ) $u_1 = v$
(f) $u_2 = v'$
(s) $u_3 = v'' - \alpha^2 v$
(g) $u_4 = u_3'$

$(\alpha U - \omega)(u_3) - \alpha U'' u_1$
$+ \frac{i}{Re}\left[u_4' + \alpha^4 u_1\right]$

$$\Rightarrow \quad u_1' - u_2 = 0$$
$$u_2' - \alpha^2 u_1 - u_3 = 0$$
$$u_3' - u_4 = 0$$
$$u_4' + Re\, i\left[(\alpha U - \omega) u_3 - \alpha U'' u_1\right] + \alpha^4 u_1 = 0$$

$$\Rightarrow \quad u_1' - u_2 = 0$$

$$u_2' - \alpha^2 u_1 - u_3 = 0$$

$$u_3' - u_4 = 0$$

$$u_4' - \alpha^2 u_3 - i\,Re\left[(\alpha U - \omega) u_3 - \alpha U'' u_1 \right]$$

Boundary Conditions

$$y = 0: \quad u_1 = 0 \quad , \quad u_3 = 1$$

$$y = y_e \text{ or } \delta: \quad u_4 + \xi u_3 = 0 \qquad\qquad \xi^2 = \alpha^2 + iR(\alpha U - \omega)$$

$$u_2 + \frac{u_3}{\alpha + \xi} + \alpha u_1 = 0.$$

R consideration perturbations must vanish at $y = \delta$

- Drop $v(0) = 0$ for $v''(0) = 1$

Discretize the above system using the trapezoidal rule to obtain a tridiagonal coefficient matrix – 4×4 blocks with complex entries (8 equations) at each j. This system is non-linear (α^2, α^4, etc) so we use Newton-Raphson to solve

- Global variable — α
- Constraint — magnitude of v – example $\int |v|\, dy = 1$ (normalization)

$$v''(0) = 1 \quad \text{and drop} \quad v'(0) = 0$$

→ impose a non trivial solution, vary parameters in appropriate combinations to satisfy $v'(0) = 0$

Spatial problem

α given $u(y/\theta, H)$

Choose Re, ω_r $\rightarrow$ solve for α

$$\left.\begin{aligned} \alpha^*_{real}(R, \omega^*, H) &= \alpha_{real}\,\theta \\ \alpha^*_{imag}(R, \omega^*, H) &= \alpha_{imag}\,\theta \end{aligned}\right\} \text{non-dimension}$$

$$\uparrow$$
$$u(y/\theta, H)$$

$$R = \frac{u_e \theta}{\nu} = Re_\theta$$

$$\omega^* = \frac{\omega \theta}{u_e}.$$

— x —

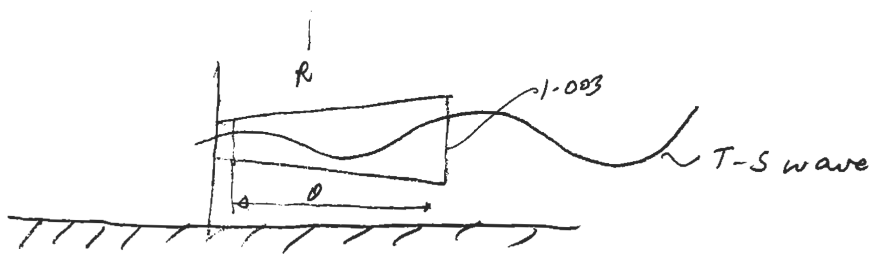
for ls handout

$$\hat{v}(y), \quad u(y) = \frac{i}{\alpha} v(y)$$

- $\hat{u}_i$ max at inflection point
- $H = 8$, $\alpha_i < 0$ $\rightarrow$ large perturbations
- $H = 2.5$, $\alpha_i > 0$ $\rightarrow$ decay of perturbations

For $H = 2.5$

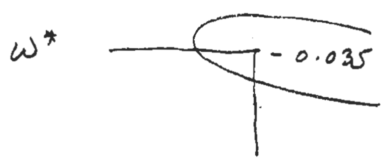
$\Rightarrow$ wave grows 0.3% in 1 θ streamwise distance



$$\lambda = \frac{2\pi}{\alpha_r} = \frac{2\pi\theta}{\alpha^*_r} \sim 630$$
$$\sim 68$$

$\theta \sim 8/10$

For $H = 3.2$



10x times growth in 1 θ distance

Briefly, note impact of suction, ht transfer, Mach # on stability (laminar)

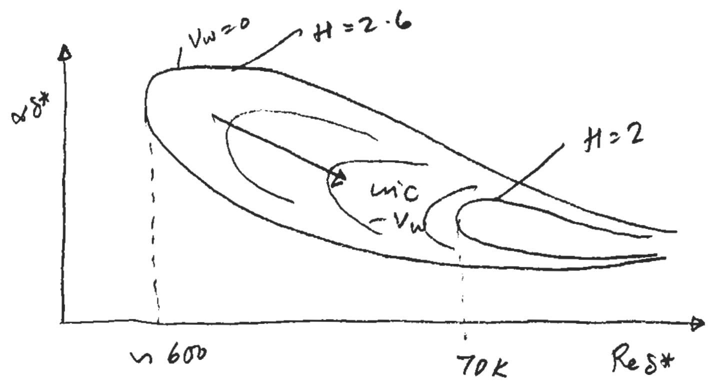
① Impact of Suction:

Stability calculation using asymptotic suction profile

$$u/U_\infty(y) = 1 - e^{V_w y/\nu}, \quad V_w < 0$$

$$\delta^* = \frac{\nu}{-V_w}$$

and $Re_{\delta^*_{crit}} = 70,000$.



Critical to minimize suction required to maintain stability

② Ht transfer: To or from the surface can enhance or lower stability limit. Ht transfer effects curvature at wall.

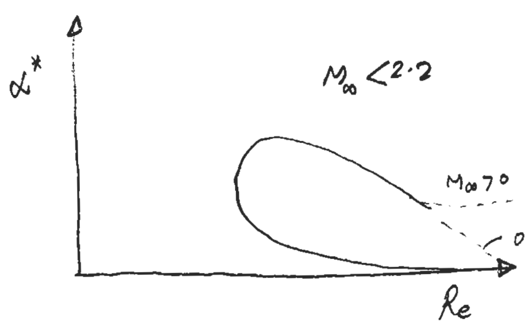
$$U''_w = -\frac{1}{\mu_w} \cdot \mu'_w \, U'_w$$

If $T_w > T_e$ (or T_∞), temperature is decreasing away from the wall $\Rightarrow \frac{\partial T}{\partial y}\Big|_w < 0$, therefore $\frac{\partial \mu}{dy}\Big|_w < 0$ $\left(\frac{d\mu}{dT} > 0 \text{ for gas}\right)$

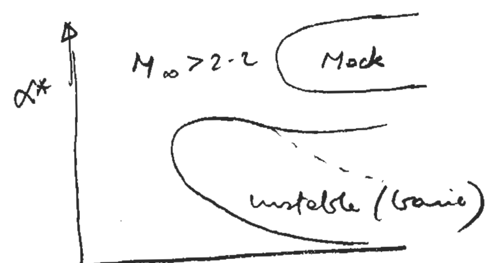
$\therefore U''_w > 0$ implying an inflection in profile

Opposite effect - stabilizing for cooling.

③ Compressibility



⇐ increases instability boundary.



$\hat{M}a^2 < 1$

local rel. M

$\hat{M}a = \dfrac{(u - c_r)}{a_\infty}$

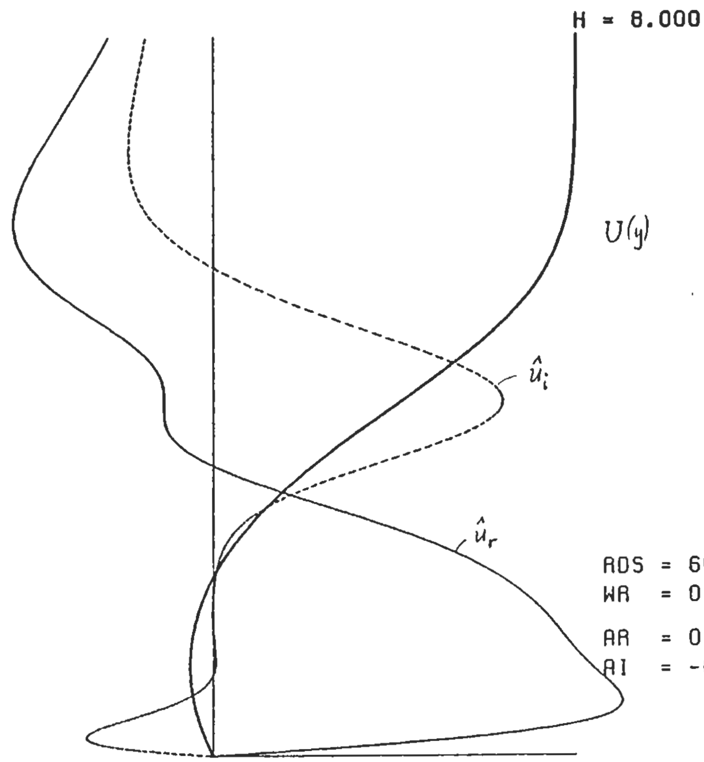
$\hat{M}a^2 > 1$ @ $M_\infty = 2.2$

$c_r \alpha_r = \omega$. $\quad c_r$ — phase velocity

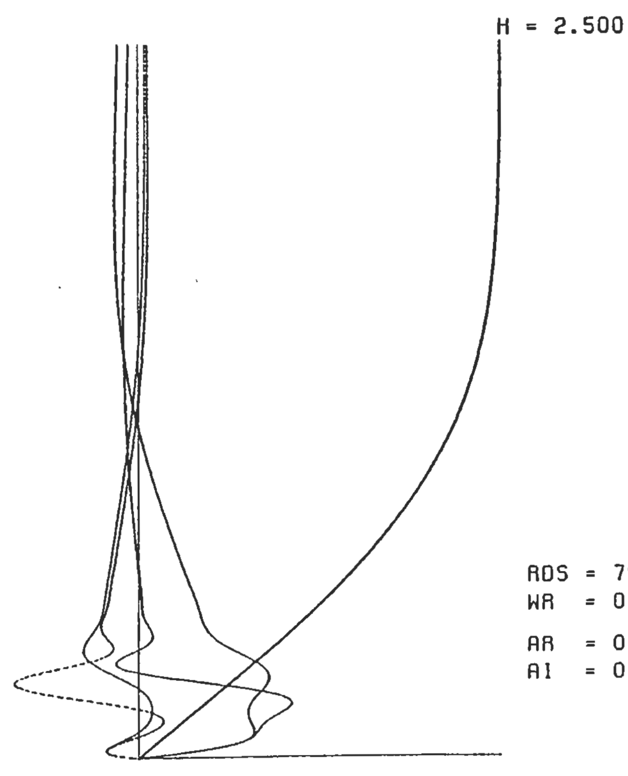
Multiple modes are due change in character of equations
- elliptical → hyperbolic

Extension of inflection point condition

$$\frac{d}{dy}\left(\rho \frac{dU}{dy}\right)\Big]_{y_s} = 0 \quad \text{at some } y_s \text{ in B-L.}$$



RDS = 640 $= Re_{\delta^*}$
WR = 0.00100 $= \omega\nu/u_e = \frac{\omega\delta^*}{u_e}\frac{1}{Re_{\delta^*}}$
AR = 0.18757 $= \alpha_r$
AI = -0.04210 $= \alpha_i$



RDS = 7905
WR = 0.00001
AR = 0.10888
AI = 0.04408