Similarity in Wind Tunnel Testing

In terms of non-dimensional force and moment coefficients, these depend on numerous non-dimensional input parameters.

$$C_L = C_L(M_\infty, \mathrm{Re}, \alpha, ...)$$
$$C_D = C_D(M_\infty, \mathrm{Re}, \alpha, ...)$$

In many aerodynamic applications,

$$C_L = C_L(M_\infty, \mathrm{Re}, \alpha)$$
$$C_D = C_D(M_\infty, \mathrm{Re}, \alpha)$$

So, to match flight condition C_L & C_D in an experiment, we should match flight M_∞, Re, and α.

* Matching α is relatively easy
* How about simultaneous M_∞ & Re matching?

$$M_\infty = \frac{V_\infty}{a_\infty} \qquad \mathrm{Re} = \frac{\rho_\infty V_\infty L}{\mu_\infty}$$

Consider the Wright Brothers Wind Tunnel:
* a_∞ is the atmospheric speed of sound at ground level to good approximation

$$\Rightarrow \qquad M_T = M_\infty$$
$$V_T = V_\infty \frac{a_T}{a_\infty}$$

For altitudes from 0-30k ft., a_∞ varies by only about ~15%.

$$\Rightarrow V_T \approx V_\infty !$$

$$a_{ok} \approx 1100 \text{ ft/sec}$$
$$a_{30k} \approx 950 \text{ ft/sec}$$

Since Wright Brothers Wind Tunnel is limited to about $V_T < 200$ mph, we can't match M_∞ unless it is low.

* For M_∞ low, say $M_\infty \lesssim 0.3$, the effects of M_∞ are small. So, in this case, we only need match Re:

$$\mathrm{Re}_T = \mathrm{Re}_\infty$$
$$\frac{\rho_T V_T L_T}{\mu_T} = \frac{\rho_\infty V_\infty L_\infty}{\mu_\infty}$$

Consider VAT @ T/O and the use of WBWT to simulate it:

$\rho_T, \mu_T, \rho_\infty$ & μ_∞ are essentially the same $\Rightarrow$ ground conditions.

$$\Rightarrow V_T L_T = V_\infty L_\infty$$

$$\frac{L_T}{L_\infty} = \frac{V_T}{V_\infty}$$

For T/O, $V_\infty \approx 200$ mph

Max V_T, $V_T \leq 200$ mph

$$\Rightarrow \frac{L_T}{L_\infty} \cong 1!$$

Transonic & Supersonic Tests

In the case where M_∞ is larger than about 0.3, we need to consider matching of M_∞ & Re in the tunnel.

What can be done?

$$M_T = M_\infty \Rightarrow \frac{V_T}{a_T} = \frac{V_\infty}{a_\infty}$$

$$\mathrm{Re}_T = \mathrm{Re}_\infty \Rightarrow \frac{\rho_T V_T L_T}{\mu_T} = \frac{\rho_\infty V_\infty L_\infty}{\mu_\infty}$$

Note: $a = \sqrt{\gamma RT}$

$\mu = \mu(T)$ ← for ideal gas

$\rho = p/(RT)$

So, V_T is largely set by M_∞ and the values of a_T, which are achievable.
Typically $V_T = O(V_\infty)$

Then, how can we match Re?

Note: we would like to reduce L_T from L_∞ to reduce model & tunnel size.

$$\Rightarrow \frac{L_T}{L_\infty} = \frac{\rho_\infty / \mu_\infty}{\rho_T / \mu_T} \frac{V_\infty}{V_T}$$

some flexibility

largely set by M_∞

$$\frac{\rho_T}{\mu_T} = \frac{p_T / RT_T}{\mu_T} = \frac{p_T}{RT_T \mu_T(T_T)}$$

One possible approach: increase pressure in tunnel.

Engineering Solutions to M_∞ & Re Matching

Traditionally:

* Test at Re_∞ but neglect M_∞ match if M_∞ is low

* Test at M_∞, trip boundary layers, ignore Re match at high Re (Re > 10^6)

* Estimate effects of not matching comp. & flight tests