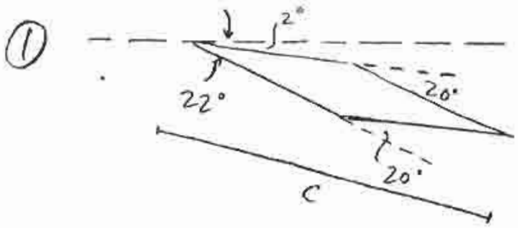
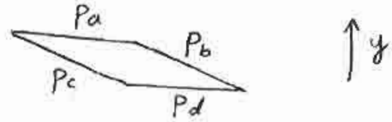
16.100 Homework #7 Solutions

① 



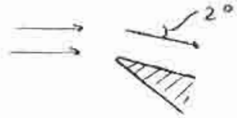
a) <u>Approach</u>: resolve forces caused by pressure in y-direction to get L', then convert to C_l.

$$C_l = \frac{L'}{\frac{\gamma}{2} p_\infty M_\infty^2 c}$$

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho (M\cdot a)^2 = \frac{1}{2}\rho\left(M\cdot\sqrt{\gamma R T}\right)^2 = \frac{1}{2}\rho\left(M\cdot\sqrt{\frac{\gamma p}{\rho}}\right)^2 = \frac{\gamma}{2} p M^2$$

To get L', need pressures p_a, p_b, p_c, p_d.

<u>Face (a)</u>: Expansion wave, since flow is turned away from itself (see Anderson).



Need to get the pressure above face (a), so we can get the Mach no. using the Prandtl-Meyer function, then get p_a.

$$\nu_a = \nu_\infty + \theta \qquad \theta = \text{turning angle}$$

$= \nu(M_\infty) + 2° \qquad \nu(M_\infty) = \nu(3) = 49.76°$ using Appendix C (Anderson)

$= 49.76° + 2°$

$= 51.76° \longrightarrow$ which Mach no. has this ν value?

Thus, $M_a = 0.31$.

Assume freestream pressure is p_∞, $\gamma = 1.4$.

$$\frac{p_a}{p_\infty} = \frac{p_a}{p_{0a}}\frac{p_{0\infty}}{p_\infty}$$ since expansion is isentropic, no stag. p chg.

$$= \left(1+\frac{\gamma-1}{2}M_\infty^2\right)^{\gamma/\gamma-1}\cdot\left(1+\frac{\gamma-1}{2}M_a^2\right)^{\gamma/1-\gamma}$$

(or just use Appendix A, isentropic flow properties)

$$= 36.73\cdot\frac{1}{42.65}$$

so $p_a = 0.8612\, p_\infty$. $\longrightarrow$

Face (b): Another expansion wave, $\theta = 20°$

(Same method as before)

$$\nu_b = \nu_a + \theta = 51.76° + 20° = 71.76°$$

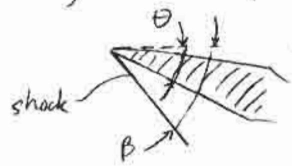
$$\rightarrow M_b = 4.5 \quad \text{using Appendix C}$$

$$\frac{p_b}{p_\infty} = \frac{p_b}{p_{0,b}} \frac{p_{0\infty}}{p_\infty} \quad \text{(no stag. pressure loss in either top expansion wave)}$$

$$= 36.73 \cdot \frac{1}{289.4}$$

so $p_b = 0.1269\, p_\infty$.

Face (c): Oblique shock wave on lower leading edge, so we expect pressure p_c to be higher than p_∞.



θ is obstruction angle $= 22°$

β is angle of shock

To get p_c, need the normal Mach number, since it controls the "jump" in flow properties. "Normal" means perpendicular to the shock (use β, not θ).

What is β? See section 9.2 for important θ-β-M relation. Using figure 9.7, knowing θ, M_∞, can get $\beta = 40°$

$$M_{\infty n} = M_\infty \sin\beta = 3 \sin 40° = 1.93$$

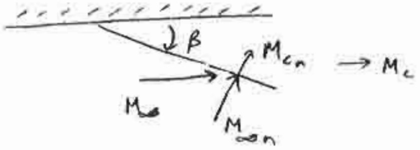
Can use normal shock table to get $\frac{p_c}{p_\infty}$ now. or use

$$\frac{p_c}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{\infty n}^2 - 1\right)$$

so $p_c = 4.179\, p_\infty$.

$\longrightarrow$

We also need the Mach no. over face (c) in order to figure out p_d after the expansion wave.



We know $M_{\infty n}$, the normal Mach no. before the shock. Using normal shock relations, we can find M_{cn}, and then use trigonometry to get M_c.

To get M_{cn}, use Appendix B or eq'n 9.14:

$$M_{cn} = \sqrt{\frac{1+\frac{\gamma-1}{2}M_{\infty n}^2}{\gamma M_{\infty n}^2 - \frac{\gamma-1}{2}}} = 0.59 \quad \text{(normal to shock)}$$

$$\text{so } M_c = \frac{M_{cn}}{\sin(\beta-\theta)} = \frac{0.59}{\sin(40^\circ - 22^\circ)} = \underline{1.91} \quad \text{(total)}$$

Anderson eq. 9.18

<u>Face (d)</u>: Expansion wave from $c \to d$, same method as before, $\theta = 20°$.

$$\nu_d = \nu_c + \theta = \nu(M_c) + 20^\circ = 23.70^\circ + 20^\circ = 43.70^\circ$$

rough interpolation from Appendix C

$$\text{so } \underline{M_d = 2.7}.$$

To get p_d: Careful! $p_{o\infty} \neq p_{oc} = p_{od}$
oblique shocks aren't isentropic.

we can get around this:

$$\frac{p_d}{p_\infty} = \frac{p_d}{p_c}\frac{p_c}{p_\infty} = \left(\frac{p_d}{p_{od}} \cdot \frac{p_{oc}}{p_c}\right)\frac{p_c}{p_\infty} = \left(\frac{1}{23.28} \cdot 6.8\right) 4.179$$

known from last part

Appendix A using M_d, M_c

$$\text{so } p_d = \underline{1.221\, p_\infty}.$$

→

Now that we have all pressures, we can find L', the lift per unit span ($=$ pressure $\cdot$ length)

Length of each face: 

$\cos 10° = \dfrac{c/2}{l}$

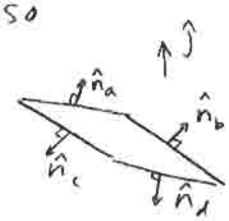
$l = \dfrac{c/2}{\cos 10°} = 0.5077c$

$L' = -\oint p\,\hat{n}\cdot\hat{j}\,ds$ where $\hat{j}$ is perpendicular to freestream $\rightarrow$ $\uparrow L'$ $\uparrow \hat{j}$

$\hat{n}$ points out of airfoil

$= -\int p_a \hat{n}_a\cdot\hat{j}\,ds - \int p_b \hat{n}_b\cdot\hat{j}\,ds - \int p_c \hat{n}_c\cdot\hat{j}\,ds - \int p_d \hat{n}_d\cdot\hat{j}\,ds$

pressures and normal vectors are constant over each face, so



$L' = -p_a(\hat{n}_a\cdot\hat{j})l - p_b(\hat{n}_b\cdot\hat{j})l - p_c(\hat{n}_c\cdot\hat{j})l - p_d(\hat{n}_d\cdot\hat{j})l$

$= -p_a l\cos 2° - p_b l\cos 22° - p_c l\cos 158° - p_d l\cos 178°$

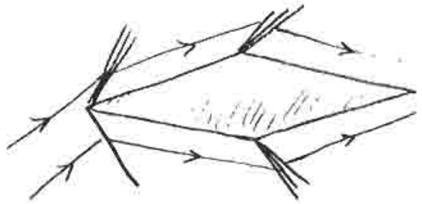
since $\hat{n}\cdot\hat{j}$'s are just cosine of angle betw. $\hat{n}, \hat{j}$

$$c_l = \frac{L'}{\frac{\gamma}{2}p_\infty M_\infty^2 c} = \frac{l}{\frac{\gamma}{2}M_\infty^2 c}\left(-\frac{p_a}{p_\infty}\cos 2° - \frac{p_b}{p_\infty}\cos 22° - \frac{p_c}{p_\infty}\cos 158° - \frac{p_d}{p_\infty}\cos 178°\right)$$

know all these values

$$= \frac{0.5077}{\frac{1.4}{2}(3^2)}(4.1166)$$

$$\boxed{c_l = 0.3317}$$

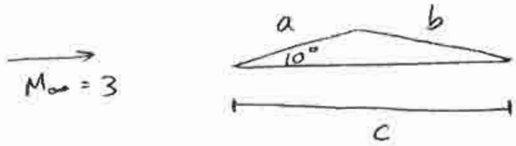
b) 

- expansion wave before face (a)
- streamlines are tangent to surfaces

⫽ = expansion

/ = shock

(2)



a) <u>Approach</u>: since pressure on lower surface is p_∞ according to shock-expansion theory, just need p_a, p_b.

<u>Face (a)</u>: $M_\infty = 3$, $\theta = 10°$, so $\beta = 27.5°$ from Fig 9.7.

Same method as in #1:

$$M_{\infty n} = M_\infty \sin\beta = 1.385$$

Using normal shock table (Appendix B), or calculating:

$$\frac{p_a}{p_\infty} = 1 + \frac{2\gamma}{\gamma+1}\left(M_{\infty n}^2 - 1\right) = \underline{2.071}$$

still need M_a for next part:

$$M_{a_n} = \sqrt{\frac{1 + \frac{\gamma-1}{2} M_{\infty n}^2}{\gamma M_{\infty n}^2 - \frac{\gamma-1}{2}}} = 0.746$$ (normal, could have used Appendix B)

$$M_a = \frac{M_{a_n}}{\sin(\beta - \theta)} = \frac{0.746}{\sin(27.5° - 10°)} = \underline{2.48}$$

<u>Face (b)</u>: Expansion wave, $\theta = 20°$ (see #1 for detailed method)

$$\nu_b = \nu_a + \theta = \nu(M_a) + 20° = 38.65° + 20° = 58.65°$$

rough Appendix C interpolation

so $M_b = 3.51$.

need p_b: $$\frac{p_b}{p_\infty} = \frac{p_b}{p_a}\frac{p_a}{p_\infty} = \left(\frac{p_b}{p_{ob}} \cdot \frac{p_{oa}}{p_a}\right)\frac{p_a}{p_\infty}$$ since $p_{oa} = p_{ob} \neq p_{o\infty}$ (isentropic expansion $a \to b$)

$$= \left(\frac{1}{77.5} \cdot 16.5\right) 2.071$$

Appendix C using M_a, M_b

so $\dfrac{p_b}{p_\infty} = \underline{0.4409}$ →

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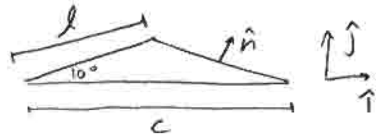
Need c_l, c_d.



$$L' = -\oint p\,\hat{n}\cdot\hat{j}\,ds \qquad D' = -\oint p\,\hat{n}\cdot\hat{i}\,ds$$

See #1 solution for detailed use of these equations. I'll skip some steps here:

since $\hat{n}\cdot\hat{j} = -1$ for bottom face

$$L' = -p_a l\cos 10^\circ - p_b l \cos 10^\circ - p_\infty c(-1)$$

$$c_l = \frac{L'}{\frac{\gamma}{2} p_\infty M_\infty^2 c} = \frac{2}{\gamma M_\infty^2}\left(-\frac{p_a}{p_\infty}\frac{l}{c}\cos 10^\circ - \frac{p_b}{p_\infty}\frac{l}{c}\cos 10^\circ + 1\right)$$

know pressure ratios, and $\frac{l}{c} = 0.5077$ from #1

$M_\infty = 3$, $\gamma = 1.4$

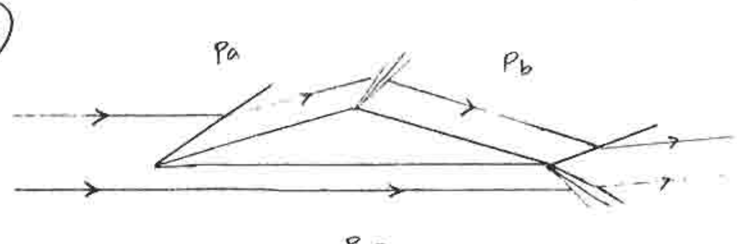
$$\rightarrow \boxed{c_l = -0.0406} \qquad \text{note: } c_l < 0\,!$$

$$D' = -p_a l \cos 100^\circ - p_b l \cos 80^\circ$$ (no contribution from lower surface, $\hat{n}\cdot\hat{i} = 0$)

$$c_d = \frac{D'}{\frac{\gamma}{2} p_\infty M_\infty^2 c} = \frac{2}{\gamma M_\infty^2}\left(-\frac{p_a}{p_\infty}\frac{l}{c}\cos 100^\circ - \frac{p_b}{p_\infty}\frac{l}{c}\cos 80^\circ\right)$$

$$\rightarrow \boxed{c_d = 0.0228}$$

b)



(Not to scale)

Note: $p_b = 0.44\,p_\infty$, so this trailing edge shock configuration is the only possibility (see following explanation)

Flow does NOT have to return to freestream direction!

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i) p_0 is NOT the same

Oblique shocks are not isentropic. There are no shocks encountered by flow below the airfoil, but there are two shocks encountered above it. Since entropy is generated above, and both upper and lower flows originate in the free stream with identical p_0, the stagnation pressure is ~~lower~~ higher (same as freestream) below the airfoil.

(Hint from 16.05: Gibbs says $T dS_0 = \underbrace{\cancel{dh_0}}_{\text{no shaft work}} - v dp_0$

$$T dS_0 = -v dp_0$$
and $dS_0 = 0$ below airfoil, so $dp_0 = 0$ also.)

ii) T_0 is the same

No shaft work is done across shocks or expansions, so both the upper and lower flows have $T_0 = T_{0\infty}$.

iii) static p is the same

If it were not, flow would accelerate sharply around the trailing edge. In this example, a shock is needed at the TE to raise the upper pressure, since $p_b = 0.44 p_\infty$, while an expansion wave helps to lower the pressure of the flow coming from below the airfoil. The flow does NOT simply return to the freestream direction.

iv) $|\vec{V}|$ is not the same

Since stagnation pressures are different for the upper and lower flows, but static pressures are equal, the Mach nos. are different. Thus, the speeds are different unless the speeds of sound above and below exactly offset the M difference to make $|\vec{V}|$ the same – not the case for this airfoil.

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