Name:.....

Massachusetts Institute of Technology

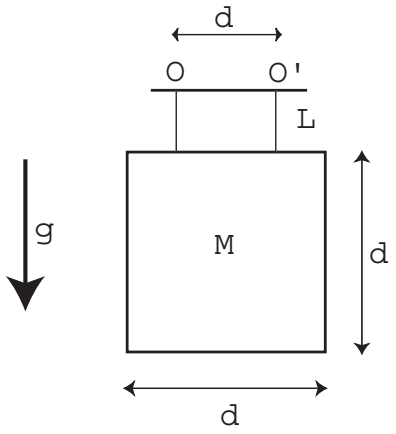
16.07 Dynamics

Final Exam

Tuesday, December 20, 2005

	Points
Problem 1 (8)	
Problem 2 (8)	
Problem 3 (10)	
Problem 4 (10)	
Problem 5 (10)	
Problem 6 (10)	
Problem 7 (10)	
Problem 8 (10)	
Problem 9 (12)	
Problem 10 (12)	
Total	

1. (8 points) The uniform square block of mass M is suspended by two massless strings of length L which suspended from the fixed points O and O'. Calculate the period of small oscillations about the equilibrium position.

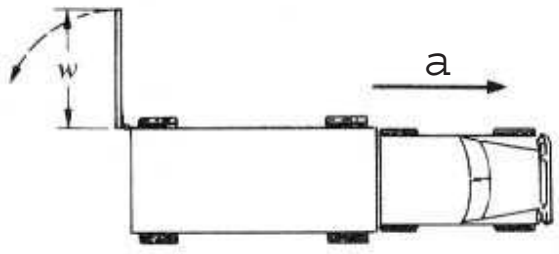


2. (8 points) A $200,000$ kg train runs east at a speed of 200 km/h at a latitude of 60^o north. Determine the **magnitude** and **direction** of the lateral force on the tracks.

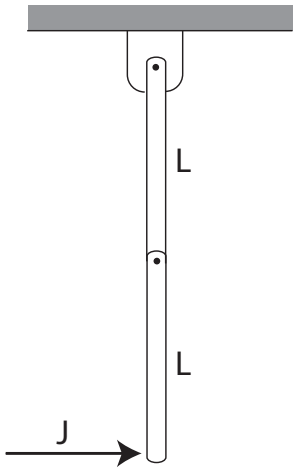
3. (10 points) A truck at rest has one door open as shown. The truck accelerates forward forward at constant acceleration a, and the door begins to swing shut. The door is uniform and solid, has total mass M, height h, and width w. Neglect air resistance

   a.- Find the instantaneous angular velocity of the door about its hinges when it has swung through 90^o.

   b.- Find the horizontal force on the door when it has swung through 90^o.



4. (10 points) The two equal slender bars each having a mass of $m = 5$ kg are suspended by frictionless hinges as shown. If a horizontal impulse of $\boldsymbol{J} = 15$ N $\cdot$ s is applied at the free end of the lower bar over a very short time interval, calculate the angular velocity of the lower bar immediately after the impact. The length of the bars is $L = 1$m.

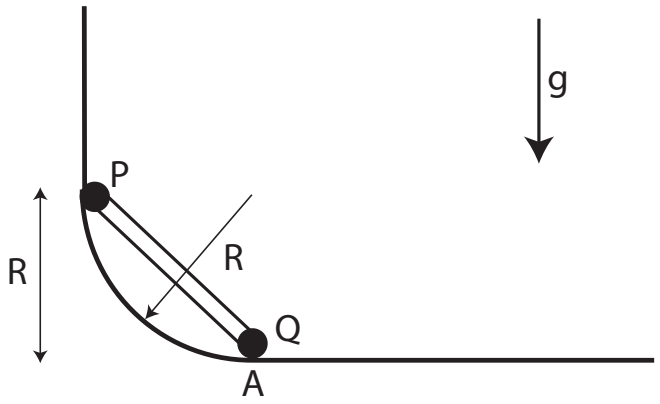


5. (10 points) The slender bar of length $\sqrt{2}$ and mass m is releases from rest at the position shown. The two ends of the bar are fitted with two small frictionless rollers.

   a.- What is the final velocity of the bar?

   b.- What is the reaction at the ends P and Q at the instant ***b***efore P reaches the point A where the circular path meets the horizontal floor

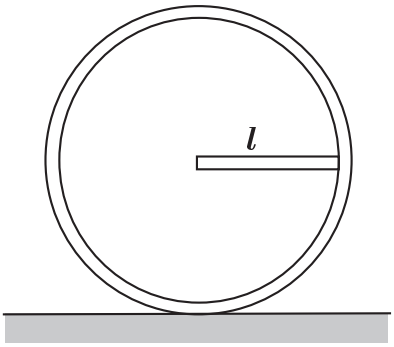
   c.- What is the reaction at the ends P and Q at the instant ***a***fter P reaches the point A where the circular path meets the horizontal floor



6. (10 points) A uniform slender rod of length l and mass m is secured to a circular hoop of radius l as shown. The mass of the hoop is M.

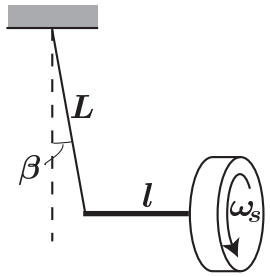
   a.- If the rod and hoop are released from rest on a horizontal surface in the position illustrated, determine the initial values of the friction and normal forces from the flat surface on the hoop assuming that friction is sufficient to prevent slipping.

   b.- Determine the period of small oscillations about the equilibrium position.



7. (10 points) A gyroscope wheel is at one end of an axle of length l. The other end of the axle is suspended from a string of length L. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass M and moment of inertia about its center of mass I_0. Its spin angular velocity is ω_s. Neglect the mass of the shaft and of the string.

   Find the angle β that the string makes with the vertical. Assume that β is so small that approximations like $\sin\beta \approx \beta$ are justified.



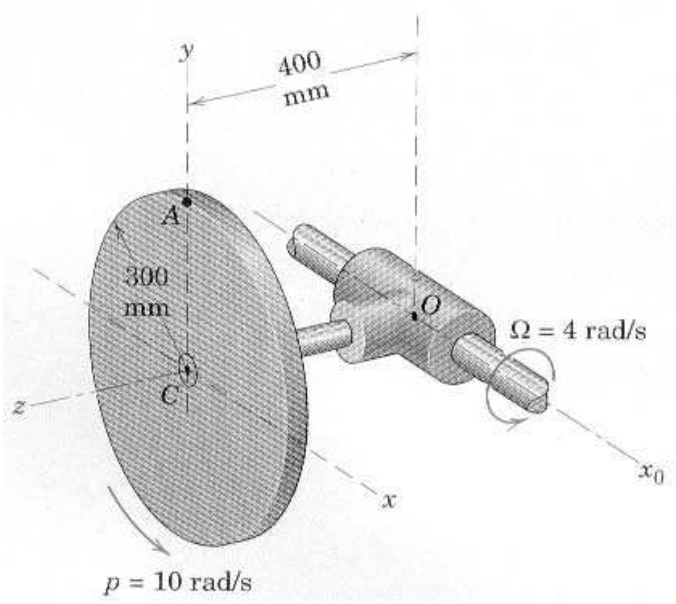
8. (10 points) A fighter aircraft has a single gas turbine engine. The rotor consists of five compressor stages and two turbine stages connected by a shaft. The total mass of the rotor is 220 kg and the radius of gyration is 0.5 m. The rotational speed is $16,000$ RPM (revs. per minute) clockwise viewed from the rear. The plane makes a $5g$ turn to the right at a constat speed of 600 km/h. What is the torque that must be applied by the aerodynamic forces to the plane to maintain horizontal flight?

9. (12 points) The uniform disk below has a mass m and is spinning with constant angular velocity $p = 10$ rad/s and simultaneously rotating about the x_0 axis with constant angular velocity $\Omega = 4$ rad/s. Ignoring any gravitational effects, we want to determine for instant shown,

   a.- the angular velocity vector and acceleration vectors of the disk

   b.- the velocity and acceleration of point A on the disk with respect to non rotating fixes axis

   c.- the moments that must be applied in the z and x_0 directions to obtain this motion (*Assume that there is no friction, the mass of the supports is negligible compared to the mass of the disk, and ignore any gravity effects*).



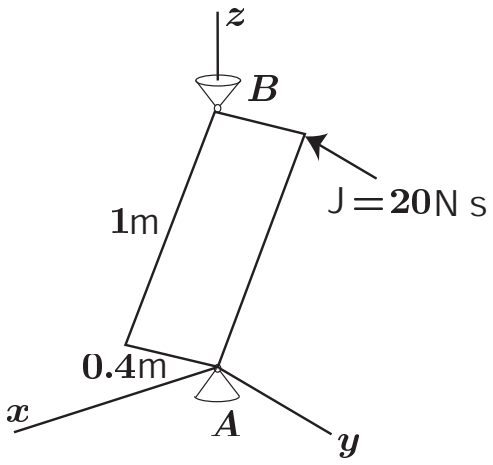
10. (12 points) A thin uniform rectangular plate of mass 5 kg is supported by to joints which allow it to spin freely about the z axis. The plate is initially at rest and suddenly an impulse of $\boldsymbol{J} = -20\boldsymbol{j}$ N·s is applied over a very short time interval to the top free corner of the plate as shown. We want to determine:

    a.- The tensor of inertia of the plate in the axes shown

    b.- The angular velocity vector of the plate after impact

    c.- The angular momentum vector and kinetic energy of the plate after impact

    d.- The impulsive reactions at the support during impact



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