Lecture S19 Muddiest Points

General Comments

Today, we began our discussion of sampling theory. We found that if we sample a band-limited signal at more than twice its bandwidth, then we can reconstruct the signal exactly. That's great, because it tells us how to design systems, such as CD players, that faithfully reproduce the original signal.

Responses to Muddiest-Part-of-the-Lecture Cards

(13 cards)

1. ***I'm going to be honest — what is a Fourier transform? How does it differ from Laplace, and when do we use Fourier? I should understand by now, but I don't get it. (1 students)*** At the most basic level, a FT is the same as a LT when the signal (say, $g(t)$) is stable. When $g(t)$ is unstable, there is no FT, although there may be a LT. However, for some signals (such as sinusoids), there is, strictly speaking, no FT or LT, because there is no region of convergence for which the LT converges. However, we can define a FT, if we do so carefully, in a limiting sense. This is good, because many signals of interest in communications (such as sinusoids) could not be analyzed by transform methods otherwise.

2. ***Basically with CDs, you're saying we sample at twice the rate of the signal, so we know the entire signal, because it's linear? [There is a figure that basically shows the sample points, with straight line interpolation in between the points.] (1)*** No, it's more subtle than that. The point is that if you sample a band-limited signal, then the samples are enough to reconstruct the signal, so long as the sample rate is greater than twice the bandwidth. But the original signal is not found by a linear interpolation between the points.

3. ***I've done some work with audio recording, and there is an option to record at 48 kHz. What benefit would there be to sample at a higher rate? (1)*** The Audio Engineering Society has a set of standards for recording. Quoting from the standards document, *AES recommended practice for professional digital audio — Preferred sampling frequencies for applications employing pulse-code modulation*,

   > A sampling frequency of 48 kHz is recommended for the origination, processing, and interchange of audio programs employing pulse-code modulation. Recognition is also given to the use of a 44,1-kHz sampling frequency related to certain consumer digital audio applications, the use of a 32-kHz sampling frequency for transmission-related applications, and the use of a 96-kHz sampling frequency for applications requiring a higher bandwidth or more relaxed anti-alias filtering.

   Basically, 48 kHz is preferred, because it reduces the requirements on antialiasing filters. We'll talk a little about this.

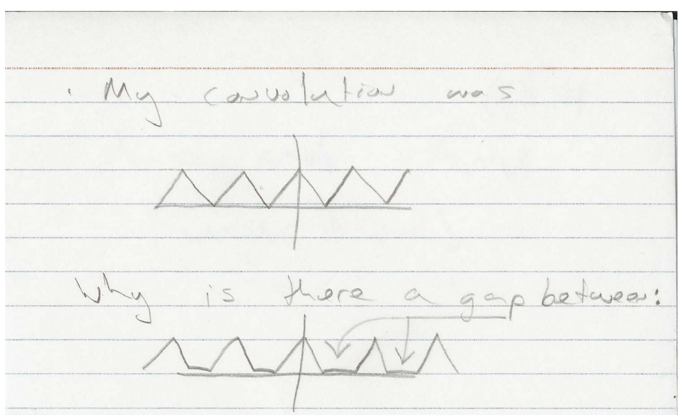
4. ***I wouldn't have fly day the day before a test and a 16.05 problem set. (1)*** Understood; we'll try to deconflict flying a little bit more from assignments next year. However, it is very difficult to due. We have to fly near the end of the term (due to weather and other considerations), and there's always a lot due near the end of term.

5. ***You said that the sampling frequency must be greater than 2 times the bandwidth of the sampled signal. How do you determine how much greater? (1)*** In theory, any sampling frequency strictly greater than twice the sampling frequency is enough. In practice, we need the sampling to be greater than that, for two reasons: First, because the low-pass filter used in the signal reconstruction is never ideal, we need some extra margin (in frequency) to allow the low-pass filter to roll off. Second, the original signal may not be truly band-limited. To make sure that aliasing does not occur (we'll talk about this next lecture), anti-aliasing filters are used to ensure that the sampled signal is band-limited. These filters are also low-pass filters, so they are not ideal, either.

6. ***What problems do arise in sampling that prevent the reconstruction from being perfect. (1)*** See above. Also, the samples are almost always digital, and there is always some error in the sampling, do to the number of bits used.

7. ***If a signal is not band-limited, then it cannot be reconstructed exactly from its samples? (1)*** Right.

8. ***My convolution was [first graph]. Why is there a gap between [as in the second graph]? (1)***



The width of each triangle is (twice) the bandwidth of the sampled signal. The distance between the peaks of adjacent triangles is the sampling frequency. If the sampling frequency is greater than twice the the bandwidth, then there is a gap between triangles. This is the whole point — we want there to be same gap.

9. ***Why are the triangles in the spectrum $Y(f)$ the same height? (1)*** Because the transform of $w(t)$ is

$$W(f) = \sum \frac{1}{T}\delta(f - m/T)$$

That is, each impulse in the spectrum $W(f)$ has the same area. But it is this transform that is convolved with $X(f)$ to produce $Y(f)$. So each replica of $X(f)$ is identical, just shifted in frequency.

10. ***I don't care anymore! (1)*** I'd be interested in knowing why you feel that way. I know it's a tough time of year — hang in there.

11. ***No mud. (4)***