Lecture S15 Muddiest Points

General Comments

Today, we started looking at Fourier transforms. In simplest form, FTs are just Laplace transforms with s replaced by $j\omega$. This can be done legitimately for any stable signal. But there's more to it than that — we can define the FT for *almost* stable signals, which is important for the analysis of many communications problems, such as modulation.

Responses to Muddiest-Part-of-the-Lecture Cards

(10 cards)

1. ***Is the Fourier transform at all related to Fourier series? (1 student)*** Yes! We got at Fourier transforms via Laplace transforms, but we could have gotten to them via Fourier series. Briefly, Fourier series can represent periodic series, with period, say, T. If we let the period T go to infinity, the Fourier series summation becomes the Fourier transform integral.

2. ***Direct vs. indirect transforms? (1)*** The direct transform is

$$\mathcal{F}[\}(\sqcup)] = \int_{-\infty}^{\infty} \}(\sqcup)\rceil^{-|\omega\sqcup}\lceil\sqcup$$

The inverse (not indirect) transform is

$$\mathcal{F}^{-\infty}[\mathcal{G}(|\omega)] = \int_{-\infty}^{\infty} \mathcal{G}(|\omega)\rceil^{+|\omega\sqcup}\lceil\omega$$

3. ***Can you explain what this means: If***

$$\mathcal{F}[\}(\sqcup)] = \mathcal{G}(|\omega) = \hat{\mathcal{G}}(\sqsupseteq)$$

then

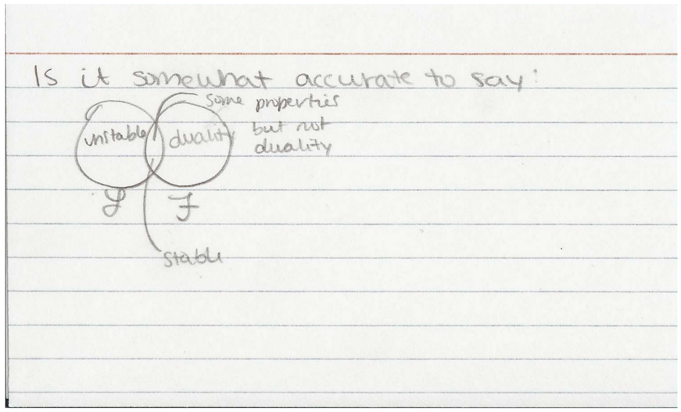
$$\mathcal{F}[\hat{\mathcal{G}}(\sqcup)] = \in\pi\}(-\omega)$$

(1) Basically, it says that if G is the FT of g, then g is the FT of G, almost, because the FT and inverse FT are almost the same thing. To make it work exactly, we have to keep track of the slight difference between the definition of the FT and the inverse FT.

4. ***What does $t \to T + t$ mean? (1)*** It means to replace t everywhere in the integral by $t + T$, which is just a change in variables.

5. ***The lecture was very muddy, maybe it's because we're all hosed. (1)*** Yeah, I thought so. We'll try to review at opportune moments.

6. ***Is it accurate to say [see card below]?***



(1) No, I don't think so. Here's what I would say: When a signal is stable, its FT and LT are the same, so the theories coincide. When a signal is "almost stable," the FT is still defined (in a limiting sense), but the LT may ore may not exist, and in any event, the FT is not the same as the LT. When an FT exists, though, duality properties exist which are useful, and not as clearly defined for LTs.

7. ***For either an even or odd function, won't there always be both a causal and noncausal part? and if so, how does this apply to a real-life, causal signal? (1)*** Every signal can be decomposed as a sum of an even part and an odd part, neither of which is causal. The FT is then the FT of the even part (which is real) plus the FT of the odd part (which is imaginary). So the FT of every causal signal (except an impulse at $t = 0$) has both real and imaginary parts.

8. ***Why can you ignore the real part and write***

$$\mathcal{F}[\}(\sqcup)] = \mathcal{G}(|\omega)$$

(1) I'm not sure I understand where this question comes from — please see me at office hours.

9. ***Completely lost you in duality. (1)*** We'll talk about this some more, in recitation.

10. ***Why are we always using $j\omega$? (1)*** The FT is basically the LT, with s imaginary, so s can be expressed as $j\omega$.

11. ***What constant do you add in the equation***

$$\int_{-\infty}^{t} g(\tau)\, d\tau \longleftrightarrow \frac{1}{j\omega} G(j\omega) + \text{??}$$

(1) The general result is

$$\int_{-\infty}^{t} g(\tau)\, d\tau \longleftrightarrow \frac{1}{j\omega} G(j\omega) + \pi G(j0)\delta(\omega)$$