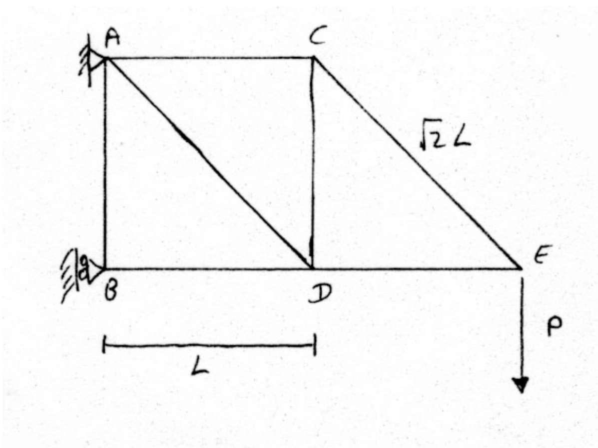
M9 Truss Deflections and Statically Indeterminate Trusses

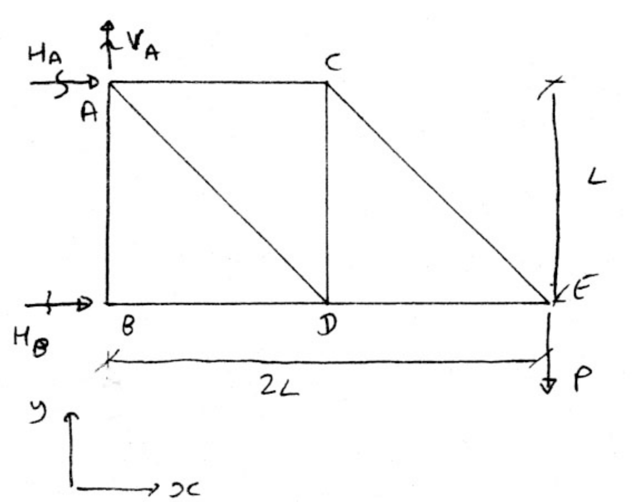
TRUSS DEFLECTION EXAMPLE

Calculate deflection of loading point E in pin-jointed truss shown below. Bars are at 90 or 45 to each other. All bars have cross sectional area A, Young's modulus E.

No temperature change occurs.



Draw FBD



$$\sum F_y \uparrow = 0 \quad V_A - P = 0$$

$$\Rightarrow V_A = P \Leftarrow \qquad (1)$$

$$\sum \overrightarrow{F_x} = 0: \quad H_A + H_B = 0$$

$$H_A = -H_B \qquad (2)$$

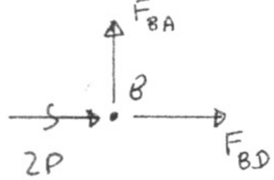
$$\sum \curvearrowright M_A = 0: \quad H_B L - 2LP = 0$$

$$H_B = 2P \Leftarrow$$

$$\Rightarrow H_A = -2P \Leftarrow$$

Analyze bar forces. Mo J.

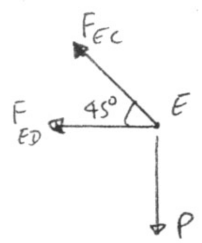
@B



$$\sum F_y \uparrow = 0 \qquad F_{BA} = 0 \Leftarrow$$

$$\sum \overrightarrow{F}_x = 0: \qquad F_{BD} + 2P = 0 \Rightarrow F_{BD} = -2P \Leftarrow$$

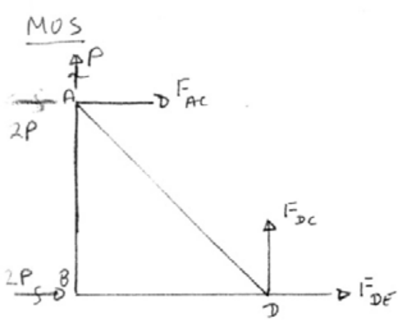
@E



$$\sum F_y \uparrow = 0: \qquad F_{EC} Sin45^\circ - P = 0 \Rightarrow F_{EC} + P\sqrt{2} \Leftarrow$$

$$\sum \overrightarrow{F}_x = 0: \qquad -F_{EC} Cos45^\circ - F_{ED} = 0$$

$$\Rightarrow F_{ED} = -P \Leftarrow$$



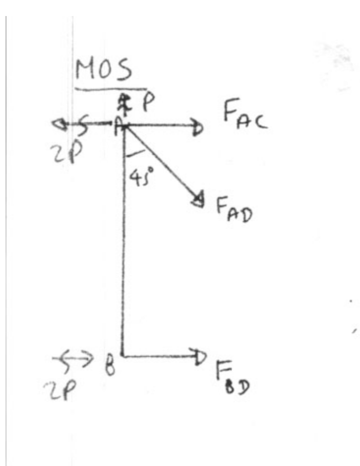
$$H_A \quad V_A$$

$$\sum \circlearrowright M_D = 0: \quad +2PL - PL - F_{AC} L = 0$$

$$\Rightarrow F_{AC} = +P \Leftarrow$$

$$\sum F_y \uparrow = 0: \quad \overset{V}{F_{DC}} + \overset{V_A}{P} = 0$$

$$\Rightarrow F_{DC} = -P \Leftarrow$$

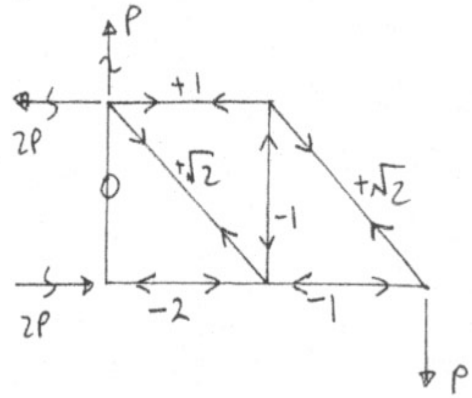


$$\sum F_y \uparrow = 0: \quad \overset{V_A}{P} - F_{AD} Cos45^\circ = 0$$

$$\Rightarrow F_{AD} = \sqrt{2} P \Leftarrow$$

Bar Deflections given by $\frac{FL}{AE}$

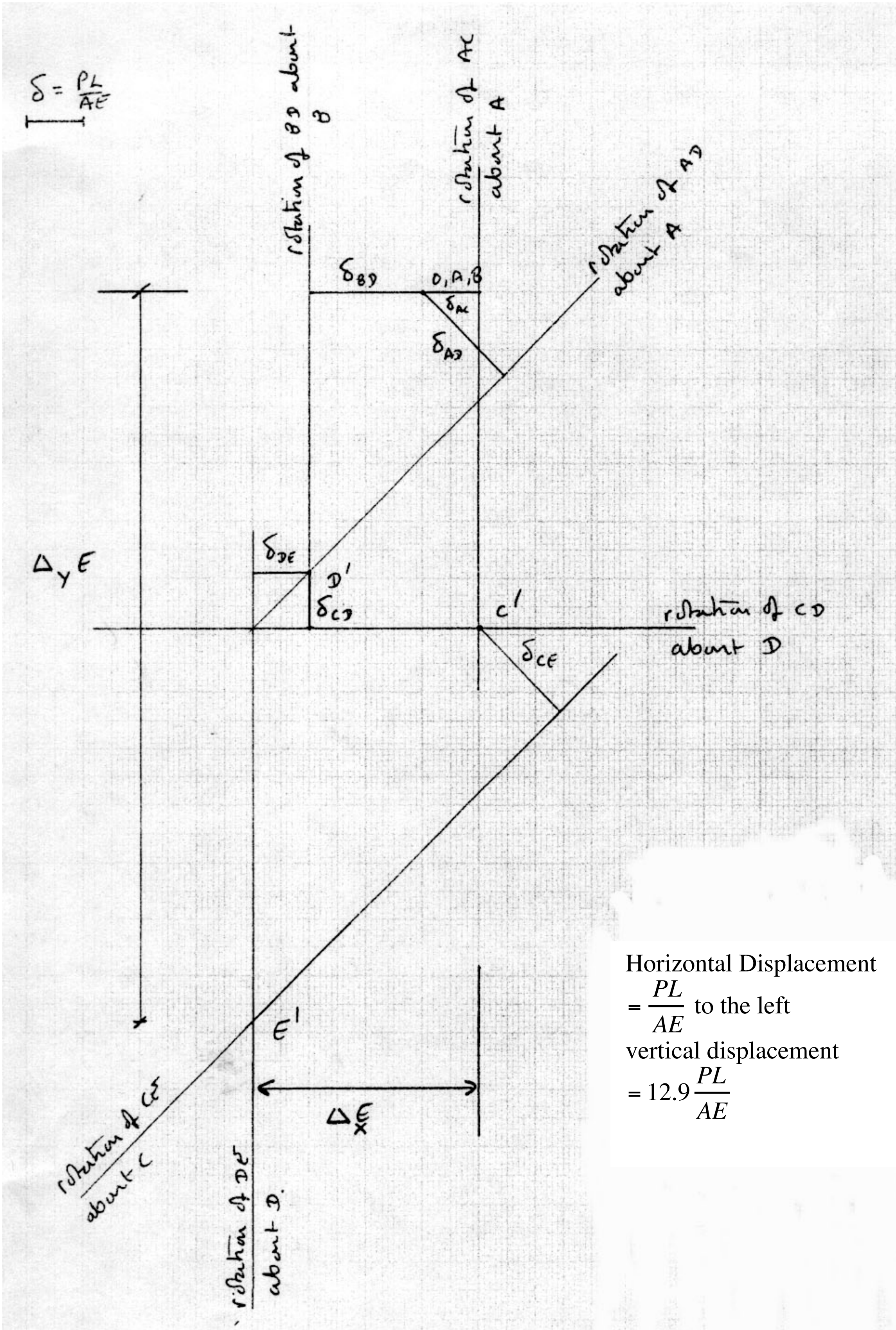
Bar	Force/P	Length/L	$\delta / \left(FL/AE \right)$
AB	0	1	0
BD	-2	1	-2
AD	$+\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
AC	+1	1	+1
CD	-1	1	-1
DE	-1	1	-1
CE	$+\sqrt{2}$	$\sqrt{2}$	$+\sqrt{2}$



Deflection Diagram:

1. Fixed points - 0, A, B
2. Locate D ' via extension/rotations of BD & AD
3. Locate C ' via extensions/rotations of AC & CD
4. Locate E ' via extensions/rotations of CE & DE

Displacement diagram (to Scale)



Horizontal Displacement

$= \frac{PL}{AE}$ to the left

vertical displacement

$= 12.9\frac{PL}{AE}$

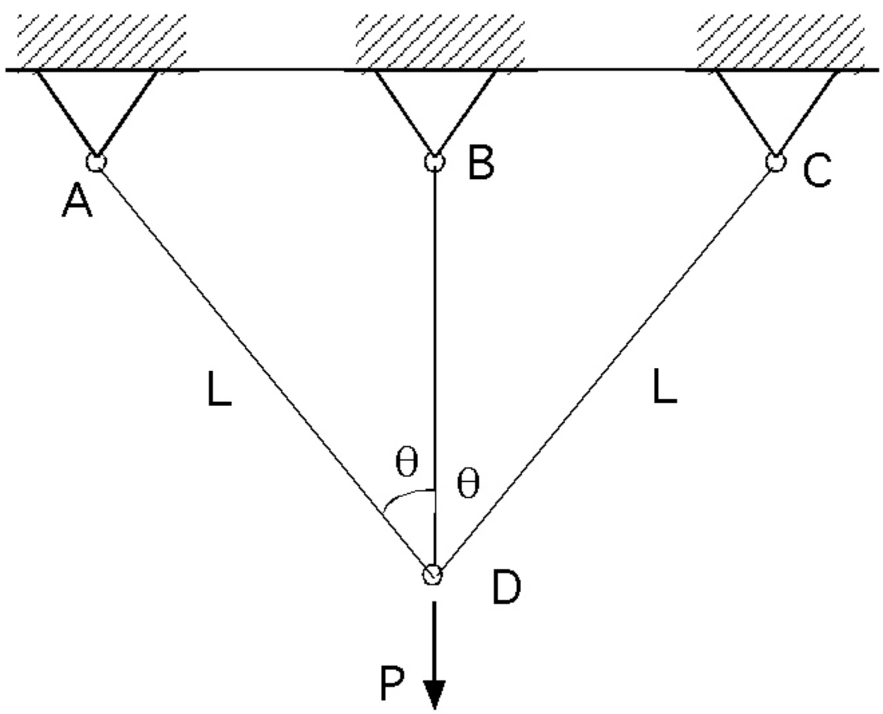
Statically Indeterminate Trusses

Can set up problem to yield a set of simultaneous equations with unknown reactions and bar forces but known displacements (at certain points - compatibility) and known constitutive behaviors

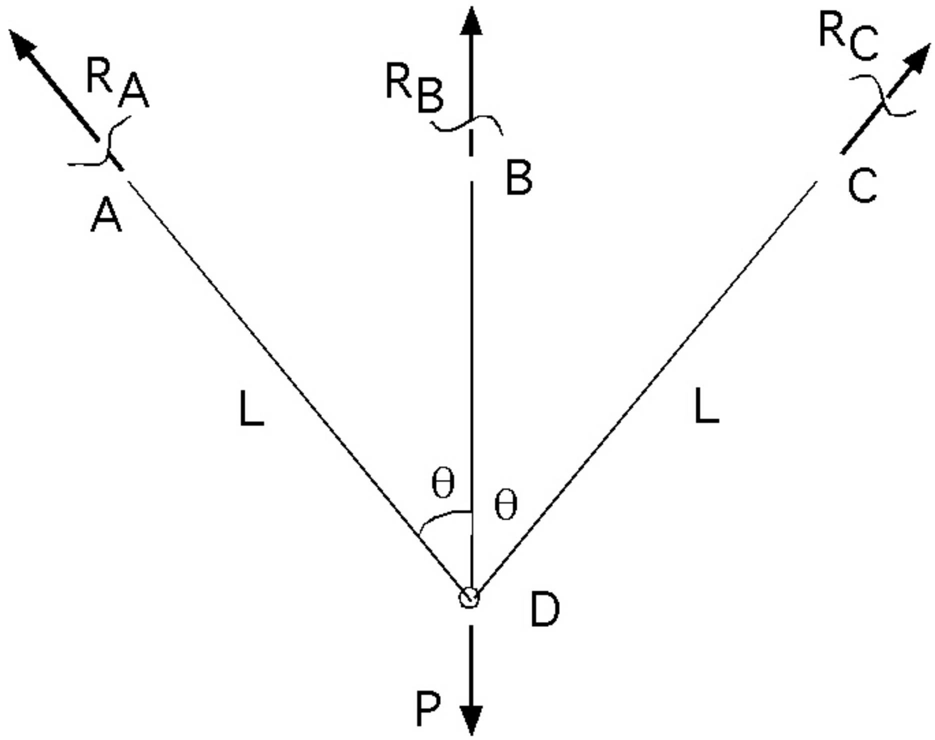
Can also use superposition and symmetry (two pretty good principles) to simplify seemingly complicated problems. Since trusses are linear (i.e if you double the applied load the internal forces and deflections will also double) we can superimpose the effects of multiple force systems in order to solve a problem.

Can extend the idea of deflection diagrams to more complicated trusses - basic principles remain the same:

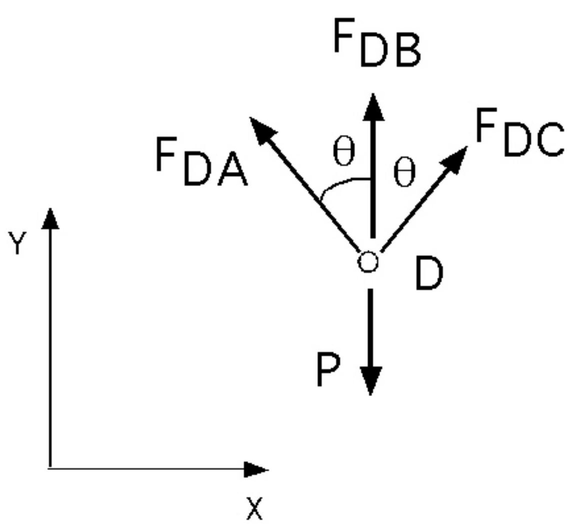
Example: Symmetric 3 bar truss, bars cross sectional area A, Young's modulus, E



FBD



or go straight to application of method of joints. *@D*



Note: $R_A = F_{DA}$, $R_B = F_{DB}$, $R_C = F_{DC}$

$\sum F_y \uparrow = 0 \qquad F_{AD}\cos\theta + F_{BD} + F_{CD}\cos\theta - P = 0$ (1.)

$\sum F_x \overrightarrow{} = 0 \qquad -F_{AD}\sin\theta + F_{DC}\sin\theta = 0$

$F_{DC} = F_{AD}$ (symmetry) (2.)

2 equations; 3 unknowns

cannot take moments - all forces pass through D

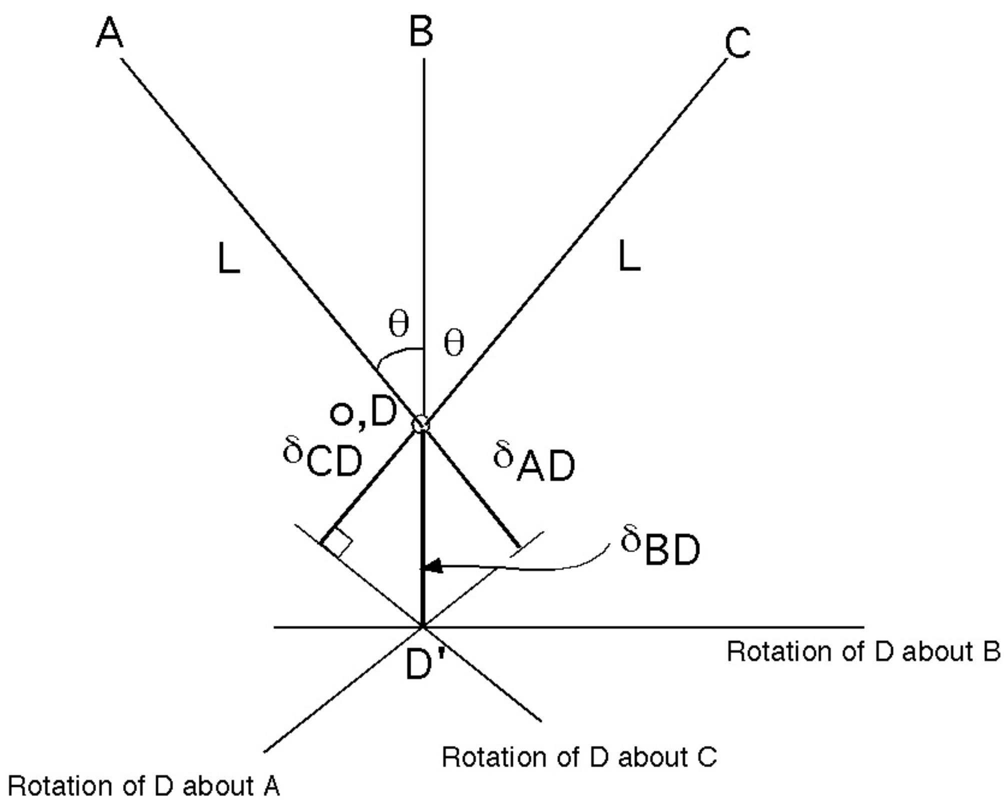
Constitutive behavior. No $\Delta T \therefore \delta = \dfrac{FL}{AE}$

Bar	Force	Length	Extension	
AD	F_{AD}	L	$\delta_{AD} = F_{AD} L / AE$	(3)
BD	F_{BD}	$L\cos\theta$	$\delta_{BD} = F_{BD} L \cos\theta / AE$	(4)
CD	F_{CD}	L	$\delta_{CD} = F_{CD} L / AE$	(5)

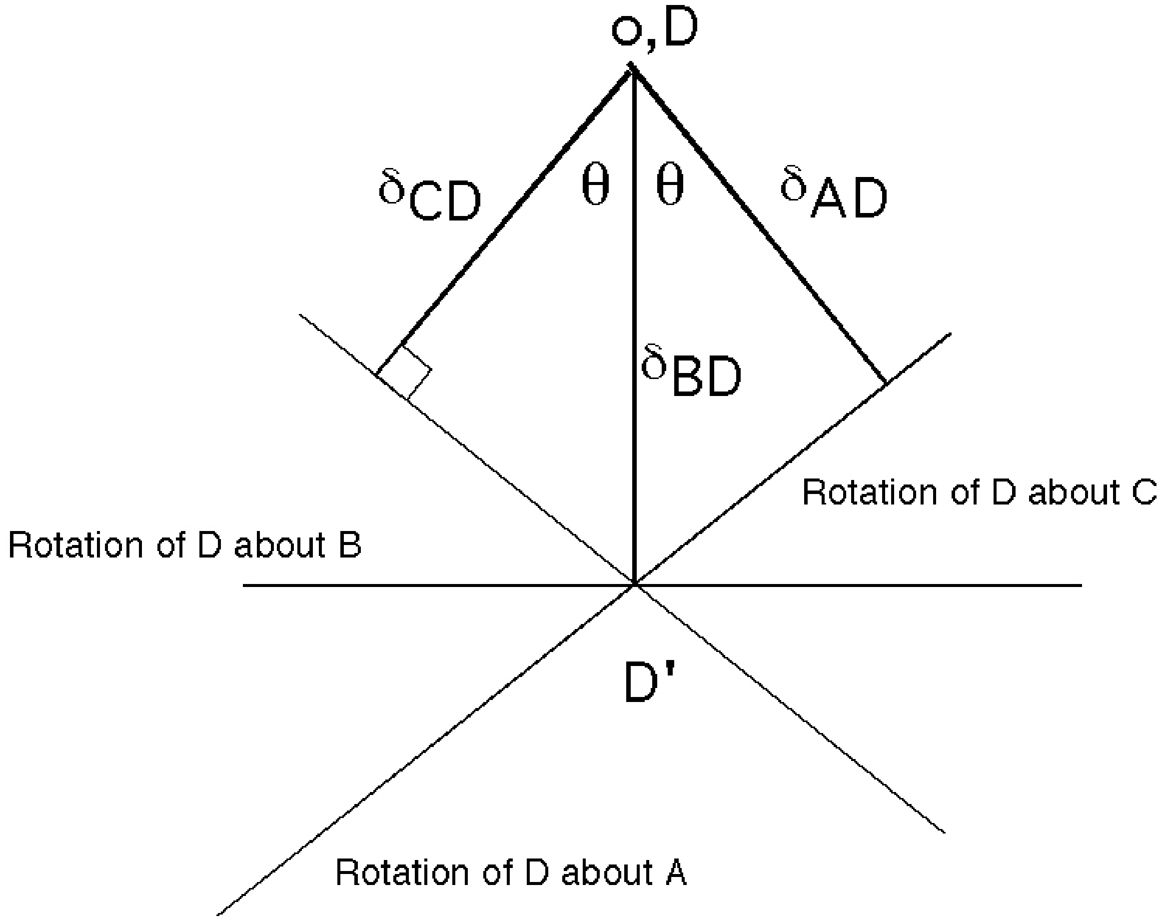
5 equations; 6 unknowns. Two equilibrium equations, 3 constitutive relations

So must invoke compatibility:

bars extend and rotate, but remain attached at *D:* Displacement diagram



Enlarged view of displacement diagram only:



$$\delta_{AD} = \delta_{CD} = \delta_{BD}\cos\theta \quad (6)$$

Now have 6 equations, 6 unknowns and can solve.

Substitute 3, 4, 5 into 6:

$$\frac{F_{AD}L}{AE} = \frac{F_{BD}L\cos^2\theta}{AE} \Rightarrow F_{AD} = F_{BD}\cos^2\theta$$

Substitute into (1)

$$2F_{BD}\cos^3\theta + F_{BD} - P = 0$$

$$F_{BD} = \frac{P}{(1 + 2\cos^3\theta)}$$

$$F_{AD} = F_{CD} = \frac{P\cos^2\theta}{(1 + 2\cos^3\theta)}$$