

Muddy Card Responses Lecture M7 2/13/2004

Synopsis. Reviewed application of simple beam theory. Introduced Macaulay's method for solving for displacements due to discontinuous loading on beams. Moved on to start to discuss the use of superposition to analyze statically-indeterminate beams. Asked concept question which is reproduced below:

M8 Concept Question 1

Two beams, are loaded, one by a symmetric distributed load, the other by a symmetric set of discrete loads. The bending moments at the center of the beam (the locations where the bending moments are a maximum) are the same. Which of the following statements are true:

A: The curvatures at the center of the beams will be the same.

B: The axial stresses at the center of the beams will be the same

C: The deflection of the center of the beams will be the same

1. A only
2. B only
3. C only
4. A and B
5. B and C
6. A and C
7. A, B and C
8. None of them
9. I do not know/I do not understand..

The correct answer is 4, namely that if the moments are the same, then the curvatures are the same. This is because the moment-curvature equation ($M = EI \frac{d^2 w}{dx^2}$) and the moment-stress equation ($\sigma_{xx} = -\frac{Mz}{I}$) are derived directly from the plan-sections remain plane assumption. Therefore they apply at a particular cross-section (x location) on the beam. So if the moments are the same then the stresses and curvatures will be the same. C is not true because the deflection of the beam at a point arises due to the deformation of the whole beam. I.e. we have to integrate the moment-curvature relationship twice, over the entire length of the beam, use the appropriate boundary conditions to obtain the constants of integration, and then evaluate the deflection at a particular point. More generally, to calculate deflections we need to integrate up strains $\epsilon_{mn} = \frac{1}{2} \left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m} \right)$ over the entire structure. Just because the strains (and hence stresses) at a point are equal it does not imply that the deflections are equal. This is a fundamental issue for all structures, so it is worth taking the time to understand.

In the PRS question (above) if two beams are geometrically identical and have identical curvatures at the center, wouldn't they have to have the same deflections at that point based solely on geometry (i.e end points are the same, could only be one possible deflection despite different loading). No. See above.

Not clear about the deflection of the centers from first PRS. Is it the same then or not? Why? No it is not, see above.

For the concept question, why does the deflection at the center of the beam remain the same? It doesn't. See above.

Could you explain the 1st PRS question in a little more detail. I understand the curvature but not the other two. See above.

If you're given the value of M at a point and not the M(x) function then you cannot find the deflection at any point w(x)? That is correct. In particular you cannot find the deflection at that particular point where you know the moment.

So is option C to PRS #1 true/not true? Please explain again. It is not true. See above.

Why was C not correct in the 1st conceptual question? No it was not true. See above.

What is the mathematical proof that the curvature is equal in the two cases? This is the derivation we did in lectures M5 and M6 going from the Euler-Bernoulli assumption that plane-sections of beams remain planar and perpendicular to the mid-plane of the beam

(centroid) during deformation. This leads directly to the moment curvature equation, which is the mathematical representation that the curvature at a point is governed by the bending moment at that point.

What was the reasoning for the beams having the same curvature in the PRS? See above.

I do not understand how axial stresses and bending moments are related. Via $\sigma_{xx} = -\frac{Mz}{I}$
- you should review the notes and make sure you understand where this comes from.

Also I am not sure what you were trying to show with the graphs. Not sure which graphs you are referring to. Please try to give me a little more to work with – even if you don't know something or are lost, give me an idea of where you stopped following.

What enables Macaulay's method to operate? The knowledge that the slope and deflection of the beam must be smooth and continuous (i.e there will be no kinks in the slope or the deflection). This means that we know that the $M(x)$ and $S(x)$ can always be integrated even if their functions are discontinuous in magnitude or slope.

For M8 PRS #2 how will we know which two to superimpose. We will discuss further in M9.

Could you explain the last PRS question's solution. We will discuss further in M9.

Is Macaulay's method always taken from the right side. I would say that I applied it from the left (in the sense that my $x=0$, origin was at the Left hand end of the beam). You can apply it from the opposite end also – it is a matter of preference. I took moments about the right hand end of the section (which I think is what you were referring to) to eliminate the shear force from the equation, so I did not have to evaluate it. I suspect that this is what you were referring to.

Can Macaulay's method be applied to any beam? Or must it be a simply supported one. No it is a general method.

I am a bit muddled on dealing with discontinuous loads and especially their deflections. It would be helpful to have more of a starting point than this. Please do come and talk to me. Also, remember that you can always divide a beam up into sections where a particular $M(x)$ function applies, and integrate each $M(x)$ function separately and match the boundary conditions of slope and deflection at the joints between the sections.

How do people with Amnesia celebrate Memorial Day? I pause for a deep groan.

There were 9 muddy cards with no mud, or positive responses. Thank you.