NAME __Solutions__

March 17, 2004

UNIFIED QUIZ 3MS
## (1-D structures handout; calculators allowed)

- Put your name on the cover page, and if you disassemble the quiz, also on each subsequent page of the exam.
- Read all questions carefully.
- Do all work on that question on that page. Use back of that page if necessary.
- Show all your work, especially intermediate results. Partial credit cannot be given without intermediate results.
- Show the logical path of your work. Explain clearly your reasoning and what you are doing. *In some cases, the reasoning is worth as much (or more) than the actual answers.*
- Be sure to show the units as well (if necessary). Intermediate answers and final answers are not correct without the units.
- Report significant digits only.
- Box your final answers.

EXAM SCORING

#1 (26%)	
#2 (25%)	
#3 (25%)	
#4 (24%)	
FINAL SCORE	

PROBLEM #1 (26%)

A pin-ended rod of length L and flexural rigidity EI is loaded with an axial compressive load. Derive (i.e. set up) the governing differential equation that will allow you to calculate the critical buckling load (**do not** solve the equation). **Note:** simply stating the equation is not sufficient to earn full credit.
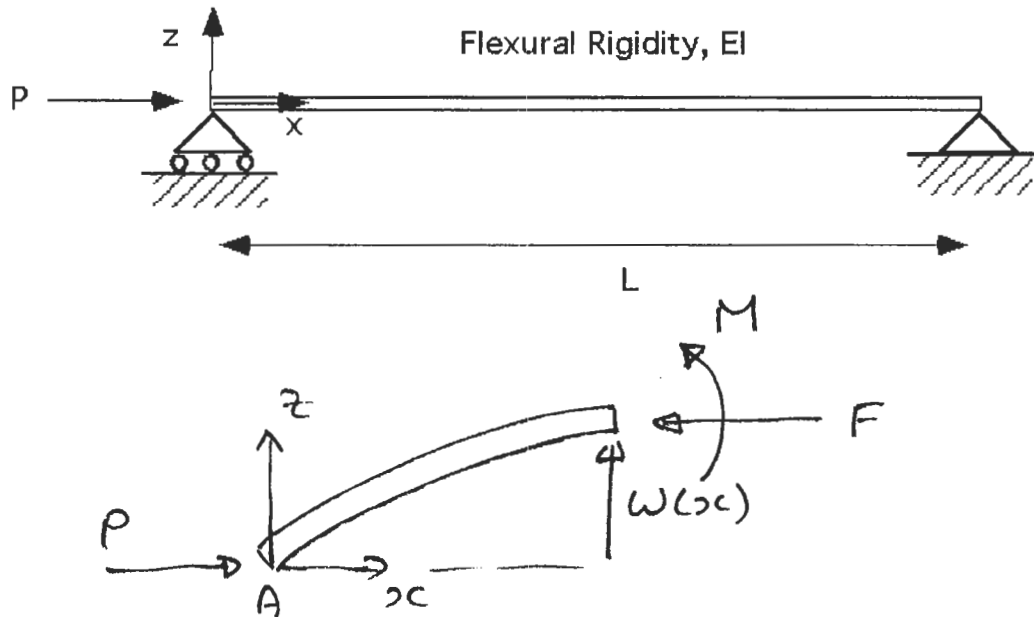
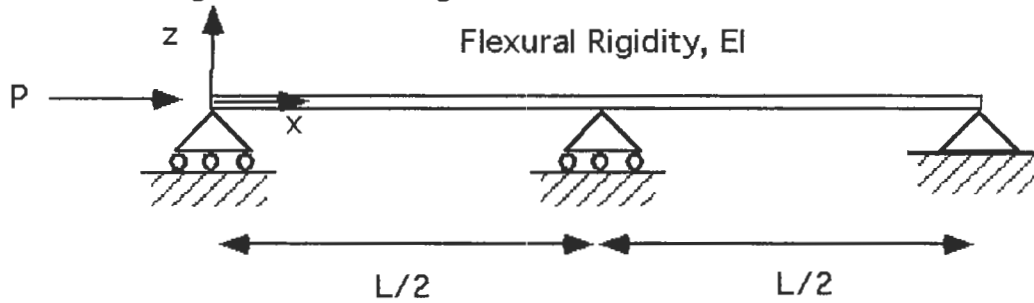$$\sum F_x \rightarrow = 0 \qquad P - F = 0 \qquad \Rightarrow \ F = P \qquad (\text{can be omitted})$$

$$\sum (M)_A = 0 : \qquad M + Pw = 0$$

$$\text{but} \quad M = EI \frac{d^2 w}{dx^2} \quad - \text{ Moment curvature relationship}$$

$$\Rightarrow \quad EI \frac{d^2 w}{dx^2} + Pw = 0 \qquad \Longleftarrow$$

$$\left(\text{or} \quad \frac{d^2 w}{dx^2} + \frac{P}{EI} w = 0 \right)$$

b) The rod is supported with an additional pin/roller support at its mid point. What is the critical buckling load for this configuration?

Flexural Rigidity, EI

central pin/roller forces column into 2nd mode

$$\therefore P_{cr} = \frac{A(2)^2 \pi^2 EI}{L^2} = \frac{4\pi^2 EI}{L^2}$$

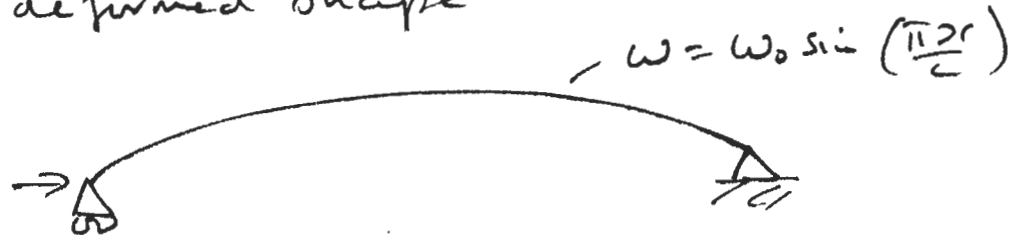or could think of buckling mode of $\frac{1}{2}$ column

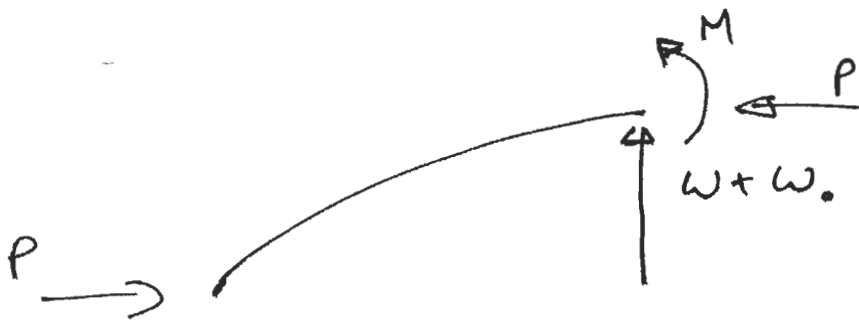$$P_{cr} = \frac{\pi^2 EI}{\left(\frac{L}{2}\right)^2} = \frac{4\pi^2 EI}{L^2}.$$

c) Discuss how your solution approach and resulting solution to part (a) would be modified if the

rod had an initial deformed shape given by $w = w_0 \sin\left(\dfrac{\pi x}{L}\right)$ where this represents a small

deviation from the straight rod of part (a). Do not solve for the deformed shape.

Initial deformed shape

$w = w_0 \sin\left(\dfrac{\pi x}{L}\right)$

obtain differential equation via

$w + w_0$

no w_0.

$$EI \frac{d^2 w}{dx^2} + (w + w_0)P = 0$$

note w_0 only enters this term

∴ need particular solution

$$\frac{d^2 w}{dx^2} + \frac{wP}{EI} = -\frac{w_0 P}{EI}$$

$$= \frac{w_0 \sin\left(\pi x / L\right)}{\left(\pi \big/ \sqrt{\frac{P}{EI}} L\right)^2 - 1}$$

Solution of form

$$w = A \sin\sqrt{\frac{P}{EI}}\,x + B\cos\sqrt{\frac{P}{EI}}\,x + Cx + D + w_{part}$$

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PROBLEM #2 (25%)

A thin walled shaft, length 3m, with a circular cross section of radius 50 mm and wall thickness 2 mm is subjected to a distributed torque of 1kNm/m acting along its length. The shaft is made of an aluminum alloy, with a Young's modulus of 70 GPa, a shear modulus of 27 GPa and a tensile yield strength of 500 MPa.

3m

t=2mm

T=1kNm/m

2R=100 mm
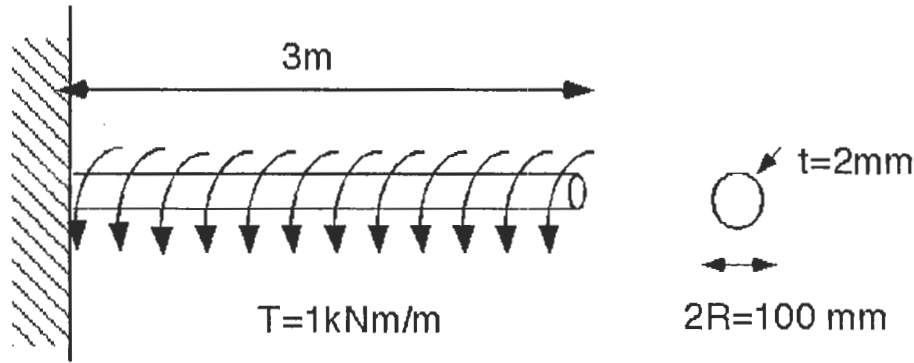
a) Calculate the maximum *shear* stress in the shaft and state its location.

Total torque at root = $3 \times 1 \, kNm/m = 3 \, kNm$

$$\tau = \frac{TR}{J} = 2\pi R^3 t \frac{T \times R}{2\pi R^2 t} = \frac{3 \times 10^3}{2 \times \pi \times (50 \times 10^{-3})^2 \times 2 \times 10^{-3}}$$

$$= 95 \, MPa \ \subseteq$$

at outer radius at root.

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b) calculate the angle of twist of the tip of the shaft.

$$T = GJ \frac{d\phi}{dx} \qquad T = 3 - 1x \qquad kNM$$

$$\phi = \frac{10^3}{GJ} \int_0^1 \underbrace{3 - x}_{T} \, dx = \frac{10^3}{GJ} \left[3x - \frac{x^2}{2} + C \right]$$

but at $x = 0$, $\phi = 0$ $\therefore$ $C = 0$

$$\therefore \quad \phi = \frac{10^3}{27 \times 10^9 \times \pi \times (50 \times 10^{-3})^3 \times 2 \times 10^{-3}} \left[3 \times 3 - \frac{9}{2} \right]$$
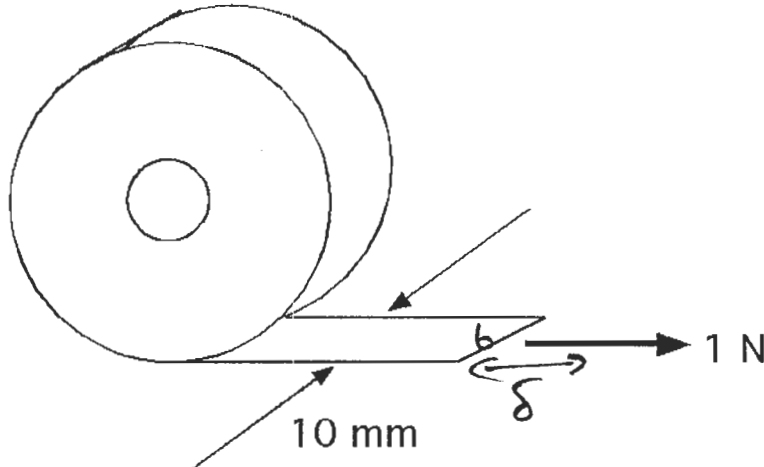
$$= 0.106 \text{ radians} \qquad (6°)$$

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PROBLEM #3 (25%)

a) I find that I can pull adhesive tape (e.g. Scotch Tape) off a roll with a constant force of 1 N. The tape is 10 mm wide. If the tape does not stretch significantly at this load estimate the toughness (in J/m^2) of the adhesive on the tape.

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at decohesion

$$dW = G_c \, dA$$

$$dU^{el} = 0 \quad \text{- no stretch in tape.}$$

$$dW = F \times \delta$$

$$dA = b \times \delta \qquad \therefore \quad F\delta = G_c \, b\delta$$

$$G_c = \frac{F}{b} = \frac{1}{10^{-2}} = 100 \ J/m^2$$

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b) For 2024 T3 aluminum alloy the fracture toughness is 44 MPa√m and the yield strength is 345 MPa. What is the maximum crack size that can be tolerated for yield to occur rather than fast fracture?

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$$K = Y\sigma\sqrt{\pi a} \qquad \text{assume } Y = 1$$
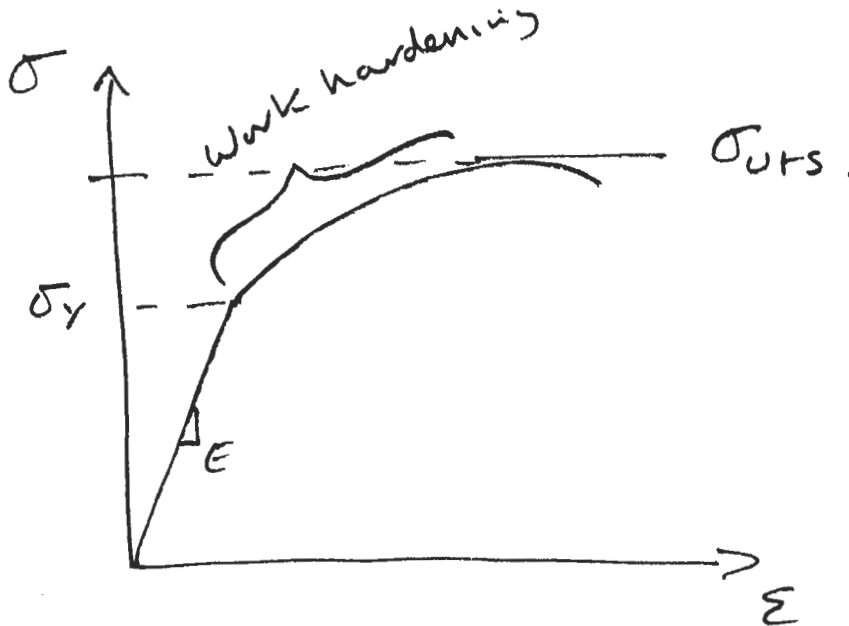
$$a_c = \frac{1}{\pi}\left(\frac{K_c}{\sigma_y}\right)^2 = \frac{1}{\pi}\left(\frac{44\times10^6}{345\times10^6}\right)^2 = \cancel{\text{5Al}}\ 5.2\times10^{-3}$$

$$= 5.2\,mm \Leftarrow.$$

PROBLEM #4 (24%)

Answer the following questions (brief, concise and accurate answers are required)

i) Sketch the stress-strain curve for a metallic alloy and indicate the yield stress, the region where work hardening occurs and the ultimate tensile strength.

ii) What mechanism(s) is(are) responsible for causing work-hardening?

Work hardening occurs due to dislocations interferring with each others motion. Can think of dislocations becoming entangled.

iii) Explain the experimental observation that low yield stress metals such as solders or pure metals are much more susceptible to fatigue failure than high strength metal alloys.

Fatigue is due to local irreversibilities, mainly due to plastic deformation in metals. Low yield stress metals undergo plasticity more easily than stronger materials and so are more susceptible to fatigue

iv) Trusses are generally found to be the most structurally efficient configuration for carrying a given load for a minimum mass structure – as can be seen by the fact that trusses are often used for spacecraft structures where aerodynamic considerations are not a concern. Why are beams less structurally efficient than trusses?.

In a truss the bars are loaded uniformly in tension or compression. i.e. All the material in a bar is exposed to the same stress. In a beam the material at the neutral axis is unloaded whereas materials at the outer ligaments $(z = \pm h/2)$ is fully loaded. If the bar and beam are both designed according to $\sigma_{max} = \sigma_{failure}$, then the beam has material that is not fully utilized for load carrying.