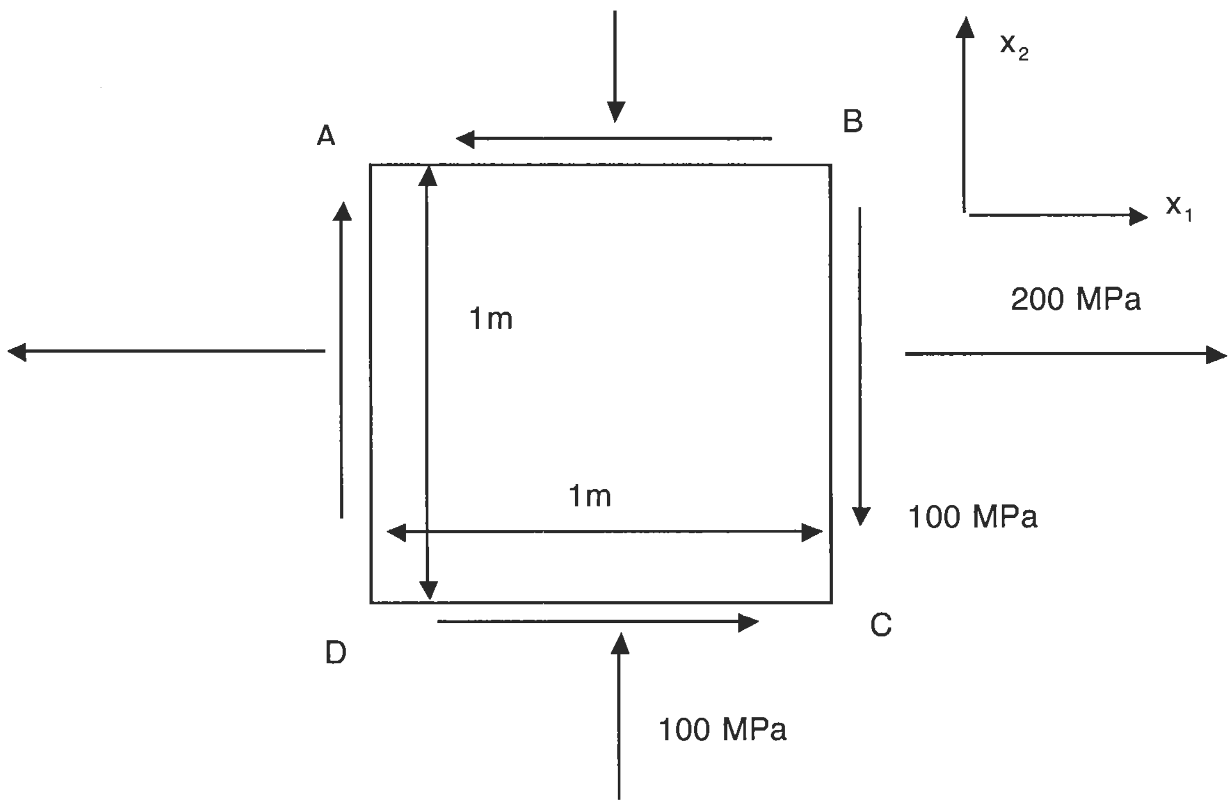
Unified Quiz 7MS
December 15th, 2003

NAME Solutions

PROBLEM #1 (25%)

a) The stress state given below is applied to an element of the aluminum alloy skin of an aircraft. Calculate the maximum shear stress and the direction(s) in which it acts. Express the direction(s) as counterclockwise angles relative to the x_1 axis.

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} 200 & -100 & 0 \\ -100 & -100 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ MPa}$$



Use Mohrs Circle

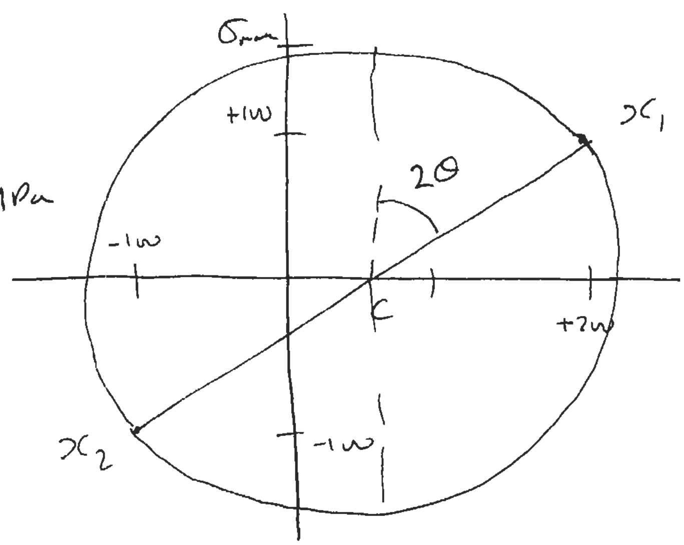
$C = +50$ MPa

$R = \sqrt{(150)^2 + 100^2} = 180$ MPa

$\sigma_{max} = R =$

$45 - \frac{1}{2}\tan^{-1}\left(\frac{100}{150}\right) = \theta = 28.2°$

and $90 + \theta = 118.2°$



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b) For the stress state in part (a) calculate the elongation (in meters millimeters or micrometers) of the diagonal, AC, of the 1m x 1m element of the wing skin. The Young's modulus of the aluminum alloy is 70 GPa and its Poisson's ratio is 0.33.

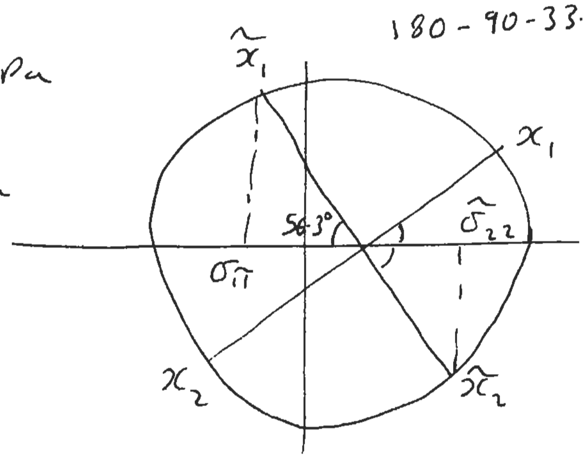
Strain @ 45° to x_1, x_2 axes

$\tan^{-1}\frac{100}{150} = 33.7^\circ$

$180 - 90 - 33.7 = 56.3^\circ$

$\tilde{\sigma}_{11} = 50 - 180\cos 56.3^\circ = -50$ MPa

$\tilde{\sigma}_{22} = 50 + 180\cos 56.3^\circ = +150$ MPa



want $\varepsilon_{\tilde{1}\tilde{1}}$

from elasticity tensor $\Rightarrow$ $\varepsilon_{11} = -\frac{50\times10^6}{70\times10^9} - \left(\frac{+150\times10^6\times0.33}{70\times10^9}\right)$

$= -1.42\times10^{-3}$

$\therefore$ elongation $= -1.42\times10^{-3}\times\sqrt{2} = -2.0\times10^{-3}$ m

$= 2$ mm

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Max Shear = 180 MPa

at 28.2° and 118.2° counterclockwise from x,

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PROBLEM #2 (30%)

a) An elastic material experiences a displacement field given by:

$$u_1 = a x_1 x_2^2$$

$$u_2 = \frac{1}{2} a \left(x_1^2 - x_2^2 \right)$$

$$u_3 = 0$$

where a is a constant.

Determine:

i) The 6 components of the strain tensor as a function of position (i.e. in terms of x_1, x_2, x_3)

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} = a x_2^2 \qquad \varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = -a x_2$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) = ~~2 a x_1 x_2^2 + a x_1~~ \frac{1}{2}\left(2 a x_1 x_2 + a x_1 \right)$$

$$\varepsilon_{13} = \varepsilon_{23} = \varepsilon_{33} = 0$$

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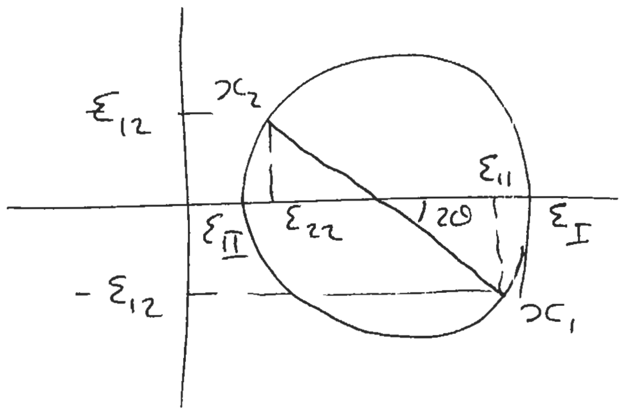
Explain how you would calculate

ii) The principal strains and the principal strain directions as a function of position.

x_3 is always a principal direction, $\varepsilon_{23} = \varepsilon_{13} = 0$

Set up Mohr's circle for in plane strain state



Solve for ε_I & ε_{II} an x_I and x_{II} by trigonometry within circle

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b) The bulk modulus, k, is the elastic constant that links volumetric strain, Δ to hydrostatic pressure p: p= k Δ

For an isotropic material, we know that only two elastic constants, E and ν, are required in order to describe the relationship between an arbitrary state of stress and an arbitrary state of strain. Derive a relationship between the bulk modulus and E and ν.

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}$$

$\sigma_1 = \sigma_2 = \sigma_3 = -P$

$\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \Delta$

$$\therefore \quad \Delta = 3\varepsilon = 3\left(\frac{1}{E} - \frac{2\nu}{E}\right)(-P)$$

$$\Delta = \frac{3}{E}(1-2\nu)(-P)$$

$$K = \frac{E}{3(1-2\nu)}$$

for $\nu = \frac{1}{3}$

$K = E$ ⟸

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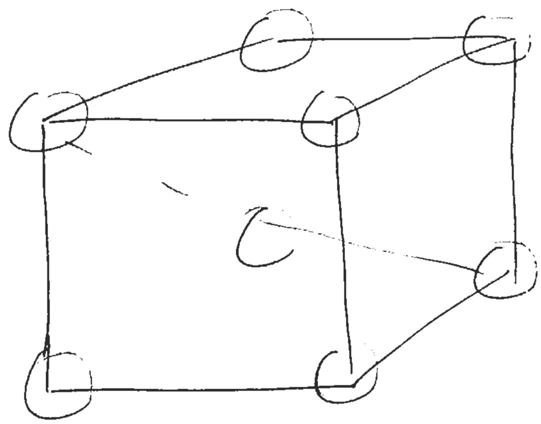
PROBLEM #3 (15%)

Molybdenum, a high melting temperature metal (T_m=2617°C), has a body centered cubic structure, an atomic radius of 0.1363 nm and an atomic weight of 95.94 g/mol. Estimate the density of the bulk material. Avogradro's number =6.023x10^{-23}



Body diagonal is close-packed

$\sqrt{3}\,a = 4r \qquad a = \frac{4}{\sqrt{3}} r$

2 atoms per cube
(1 center + 8 × $\frac{1}{8}$ corner)

$\therefore \quad \frac{\text{weight of atoms per cube}}{\text{volume of cube}} = \text{density}$

$$= \frac{\dfrac{2 \times 95.94 \times 10^{-3}}{6.022 \times 10^{23}}}{\left(\dfrac{4}{\sqrt{3}} \times 0.1363 \times 10^{-9}\right)^3} = 10.2 \times 10^3 \ \text{Kg/m}^3$$

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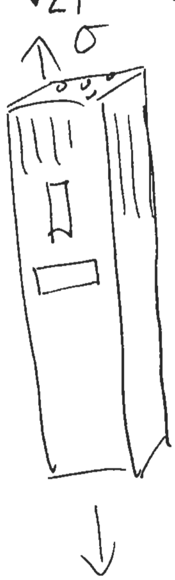
PROBLEM #4 (30%)

a) Explain why metals, in general have higher densities but lower Young's moduli than ceramics.

Metals have metallic bonding which is lower stiffness than covalent bonds commonly found in ceramics.
Metals tend to adopt close packed structures (metallic bonds are omni-directional) whereas ceramics have non close packed structures (covalent bonds are directional)
Also ceramic materials are made up of low atomic weight elements

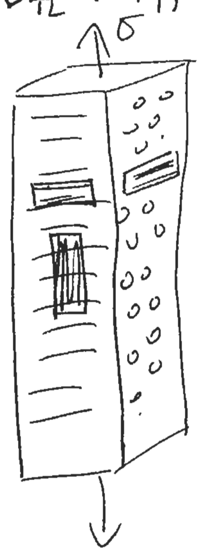
b and c – double question) You are asked to obtain the elastic constants of a unidirectional composite. Sketch the experiments you would do to obtain these quantities.

E_L + ν_{LT}



apply uniaxial load
measure longitudinal + transverse strain's

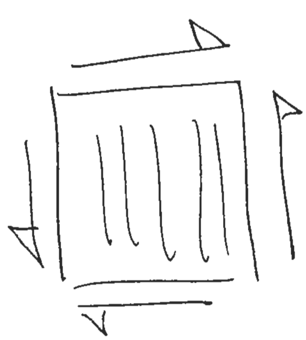
E_T, ν_{TL} + ν_{TT}



Apply uniaxial load
measure longitudinal and two transverse strains
Can calculate

$$G_{TT} = \frac{E_T}{2(\nu_{TT}+1)}$$

G_{LT} apply in plane shear



d) Carbon fiber composites with epoxy resins are stored as "pre-impregnated" plies in freezers, and are then laid up into laminates by "curing"' at an elevated temperature. The prepreg is relatively low stiffness but the cured ply will be much stiffer. Explain what is happening in the curing process.

Epoxy resins are thermoset ~~~~ polymers
Curing is the process of forming ~~~~ covalent
crosslinks between the polymer chains. It is a
thermally activated process. The uncured resin
can be stored longer at lower temperatures

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e) With reference to the material selection chart below, explain the relative suitability of Titanium, Aluminum and Steel for (i) the bars of a truss and (ii) for the wings of an airplane.

For a truss optimize $\frac{E}{\rho}$ (max)

Ti, Al, Steel are all similar

For aircraft wings, maximize $\frac{E^{1/2}}{\rho}$ – bending

Al better than ~~Stee~~ Ti better than steel

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