Problem S1 Solutions

1.a

$$x + y - 2z = -1$$
$$x + 4y + 2z = 5$$
$$x + y - z = 0$$

$$\begin{array}{r} x + y - 2z = -1 \\ x + 4y + 2z = 5 \\ \hline 2x + 5y = 4 \end{array} \qquad \begin{array}{r} x + 4y + 2z = 5 \\ 2(x + y - z) = 0 \\ \hline 3x + 6y = 5 \end{array}$$

$$\begin{array}{r} -2(3x + 6y = 5) \\ 3(2x + 5y = 4) \\ \hline 3y = 2 \end{array}$$

$$\boxed{y = 2/3}$$

$$3x + 6(2/3) = 5$$

$$3x = 1$$

$$\boxed{x = 1/3}$$

$$1/2 + 2/3 - z = 0$$

$$\boxed{z = 1}$$

1.b

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -1 \\ 1 & 4 & 2 & 5 \\ 1 & 1 & -1 & 0 \end{array}\right]$$. Subtract row 1 from row 2

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & -1 \\ 0 & 3 & 4 & 6 \\ 1 & 1 & -1 & 0 \end{array}\right]$$. Subtract row 1 from row 3

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & -1 \\ 0 & 3 & 4 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\boxed{z = 1}$$

$$\boxed{y = 2/3}$$

$$\boxed{x = 1/3}$$

1.c

$$x = \frac{\begin{vmatrix} -1 & 5 & -2 \\ 5 & 4 & 2 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -2 \\ 1 & 4 & 2 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{1}{3} \Rightarrow \boxed{x = 1/3}$$

$$y = \frac{\begin{vmatrix} 1 & -1 & -2 \\ 1 & 5 & 2 \\ 1 & 0 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -2 \\ 1 & 4 & 2 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{2}{3} \Rightarrow \boxed{y = 2/3}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & -1 \\ 1 & 4 & 5 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -2 \\ 1 & 4 & 2 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{3}{3} \Rightarrow \boxed{z = 1}$$

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

2.a

$$4x+2y+2z=7$$
$$3x+y+2z=5$$
$$x+3y-z=4$$

$$\begin{array}{r} 4x+2y+2z=7 \\ 3x+y+2z=5 \\ \hline x+y=2 \end{array} \qquad \begin{array}{r} 3x+y+2z=5 \\ 2(x+3y-z=4) \\ \hline 5x+7y=13 \end{array}$$

$$\begin{array}{r} -5(x+y=2) \\ 5x+7y=13 \\ \hline 2y=3 \end{array}$$

$$\boxed{y=3/2}$$
$$x+3/2=2$$
$$\boxed{x=1/2}$$
$$1/2+3(3/2)-z=4$$
$$\boxed{z=1}$$

2.b

$$\left[\begin{array}{ccc|c} 4 & 2 & 2 & 7 \\ 3 & 1 & 2 & 5 \\ 1 & 3 & -1 & 4 \end{array}\right]$$. Multiply row 1 by -3/4 and add to row 2

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 2 & 2 & 7 \\ 0 & -1/2 & 1/2 & -1/4 \\ 1 & 3 & -1 & 4 \end{array}\right]$$. Multiply row 1 by -1/4 and add to row 3

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 2 & 2 & 7 \\ 0 & -1/2 & 1/2 & -1/4 \\ 0 & 5/2 & -3/2 & 9/4 \end{array}\right]$$. Multiply row 2 by 5 and add to row 3

$$\Rightarrow \left[\begin{array}{ccc|c} 4 & 2 & 2 & 7 \\ 0 & -1/2 & 1/2 & -1/4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\boxed{z=1}$$
$$\boxed{y=3/2}$$
$$\boxed{x=1/2}$$

2.c

$$x=\frac{\begin{vmatrix}-1 & 1 & -2\\ 5 & 4 & 2\\ 0 & 1 & -1\end{vmatrix}}{\begin{vmatrix}1 & 1 & -2\\ 1 & 4 & 2\\ 1 & 1 & -2\end{vmatrix}}=\frac{1}{3}\Rightarrow \boxed{x=1/3}$$

$$y=\frac{\begin{vmatrix}1 & -1 & -2\\ 1 & 5 & 2\\ 1 & 0 & -1\end{vmatrix}}{\begin{vmatrix}1 & 1 & -2\\ 1 & 4 & 2\\ 1 & 1 & -2\end{vmatrix}}=\frac{2}{3}\Rightarrow \boxed{y=2/3}$$

$$z=\frac{\begin{vmatrix}1 & 1 & -1\\ 1 & 4 & 5\\ 1 & 1 & 0\end{vmatrix}}{\begin{vmatrix}1 & 1 & -2\\ 1 & 4 & 2\\ 1 & 1 & -2\end{vmatrix}}=\frac{3}{3}\Rightarrow \boxed{z=1}$$