Unified Engineering II **Spring 2004**

Problem S8 (Signals and Systems)

This problem shows why a radar system sends out a chirp, which has a broad range of frquencies in the signal, and *not* a short sinusoidal pulse, which is at a single frequency. To see why a sinusoidal pulse doesn't work well, let's try a radar signal

$$u(t) = \begin{cases} \sin(2\pi t), & -3 \le t \le 0 \\ 0, & \text{otherwise} \end{cases}$$

The matched filter for this pulse has impulse response

$$g(t) = u(-t) = \begin{cases} \sin(-2\pi t), & 0 \le t \le 3 \\ 0, & \text{otherwise} \end{cases}$$

The radar sends out a signal, $u(t)$, that reflects off the aircraft and returns to the radar system. The time it takes the signal to return is twice the distance to the aircraft, divided by the speed of light. The received signal is $u(t-T)$, where T is the round trip travel time of the signal. For the purposes of this problem, we can ignore the time delay, T, and just look at how the matched filter response to $u(t)$.

1. Find the convolution
$$y(t) = g(t) * u(t)$$
You will find it helpful to use the flip and slide method to set up the integral. The integral can be evaluated relatively easily in closed form, if you set up the integral properly.

2. Plot $y(t)$.

3. $y(t)$ as plotted above is the signal that results when the round-trip time of the pulse is zero. When the delay time is greater, of course, the signal that results is $y(t-T)$, which is just $y(t)$ shifted right by T. What feature of $y(t-T)$ would you use to identify the time T?

4. Explain why it might be difficult to determine T from a returned radar pulse, especially if there is additional noise added to the signal.

5. The signal $y(t)$ as plotted in Part 2 is called the *ambiguity function*, because it helps determine how ambiguous the delay time T is in the presence of noise. Explain why the ambiguity function corresponding to the chirp signal of Problem S6 is better than the ambiguity function in this problem.