(a)

$$x(t) = e^{-\alpha t}\, u(t)$$
$$h(t) = e^{-\beta t}\, u(t)$$

$$y(t) = h(t) * x(t)$$
$$= \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$
$$= 0, \qquad t < 0$$
$$= \int_0^t e^{-\beta(t-\tau)}\, e^{-\alpha\tau}\, d\tau, \qquad t \geq 0$$
$$= \int_0^t e^{-\beta t}\, e^{(\beta-\alpha)\tau}\, d\tau$$

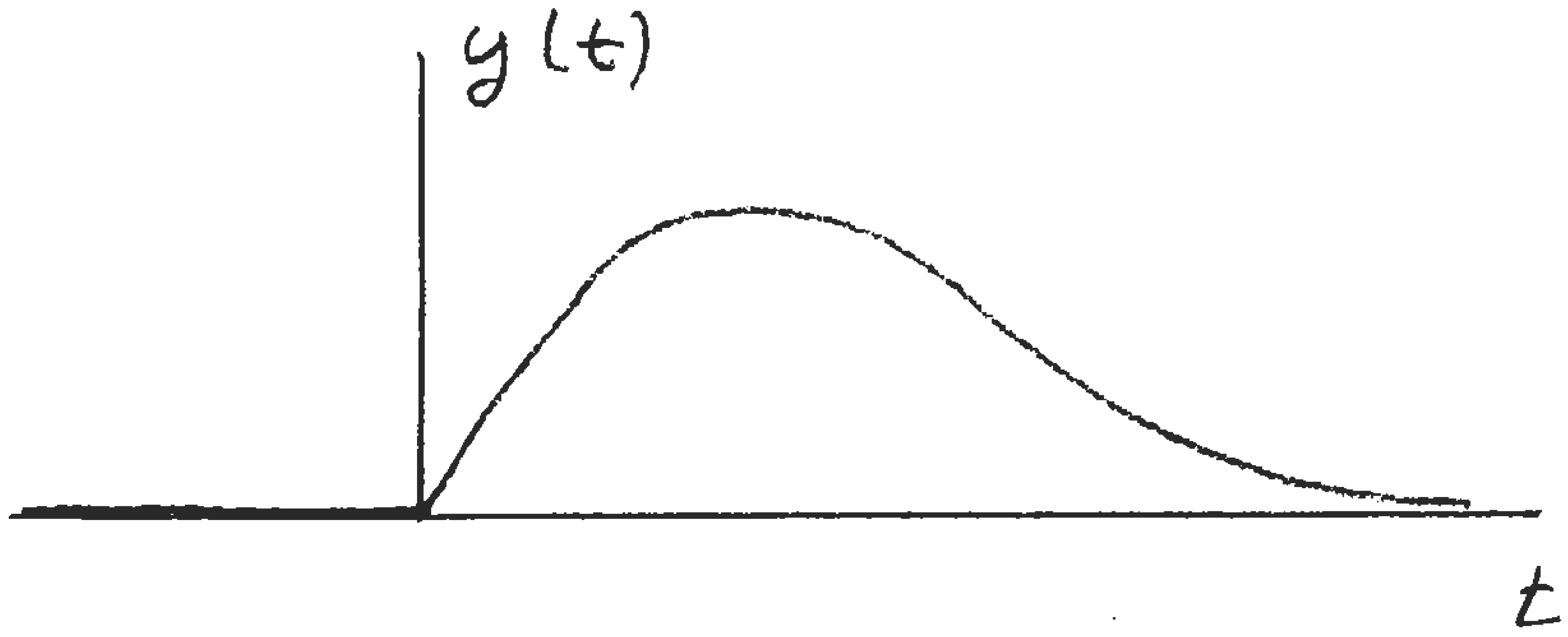
If $\beta \neq \alpha$, then

$$y(t) = \int_0^t e^{-\beta t}\, e^{(\beta-\alpha)\tau}\, d\tau$$
$$= e^{-\beta t}\, \frac{1}{\beta-\alpha}\, e^{(\beta-\alpha)\tau} \Big|_{\tau=0}^{t}$$
$$= \frac{1}{\beta-\alpha}\, e^{-\beta t}\left(e^{(\beta-\alpha)t} - 1\right)$$
$$= \frac{1}{\beta-\alpha}\, e^{-\alpha t} - \frac{1}{\beta-\alpha}\, e^{-\beta t}, \qquad t \geq 0$$

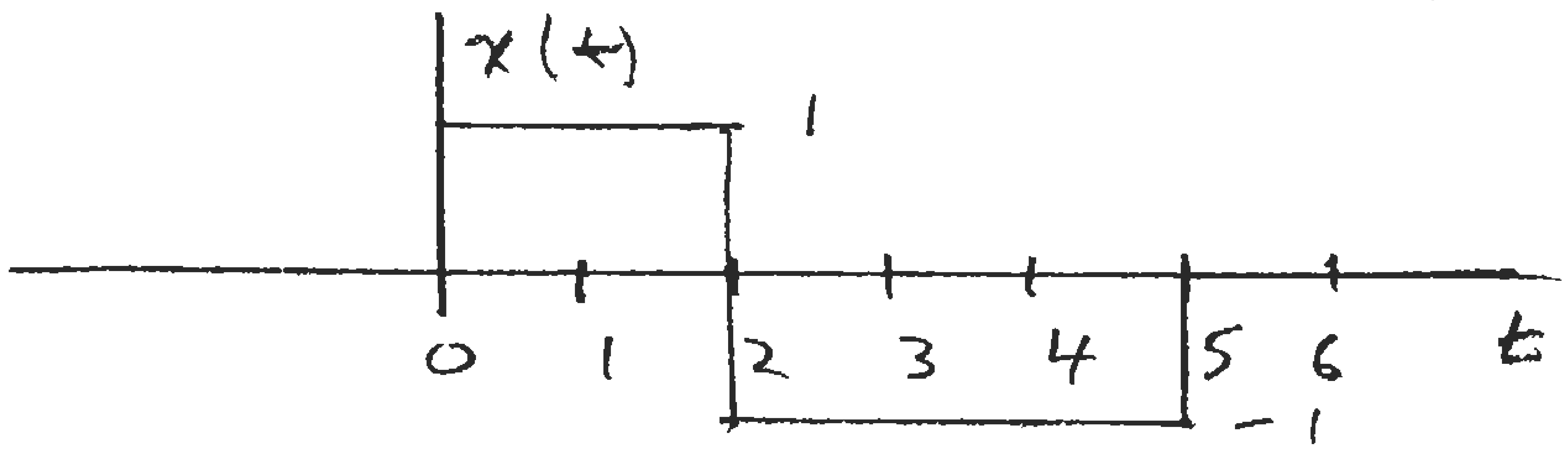
If $\beta = \alpha$,

$$y(t) = \int_0^t e^{\alpha t}\, e^{0\tau}\, d\tau$$
$$= t\, e^{\alpha t}, \qquad t \geq 0$$

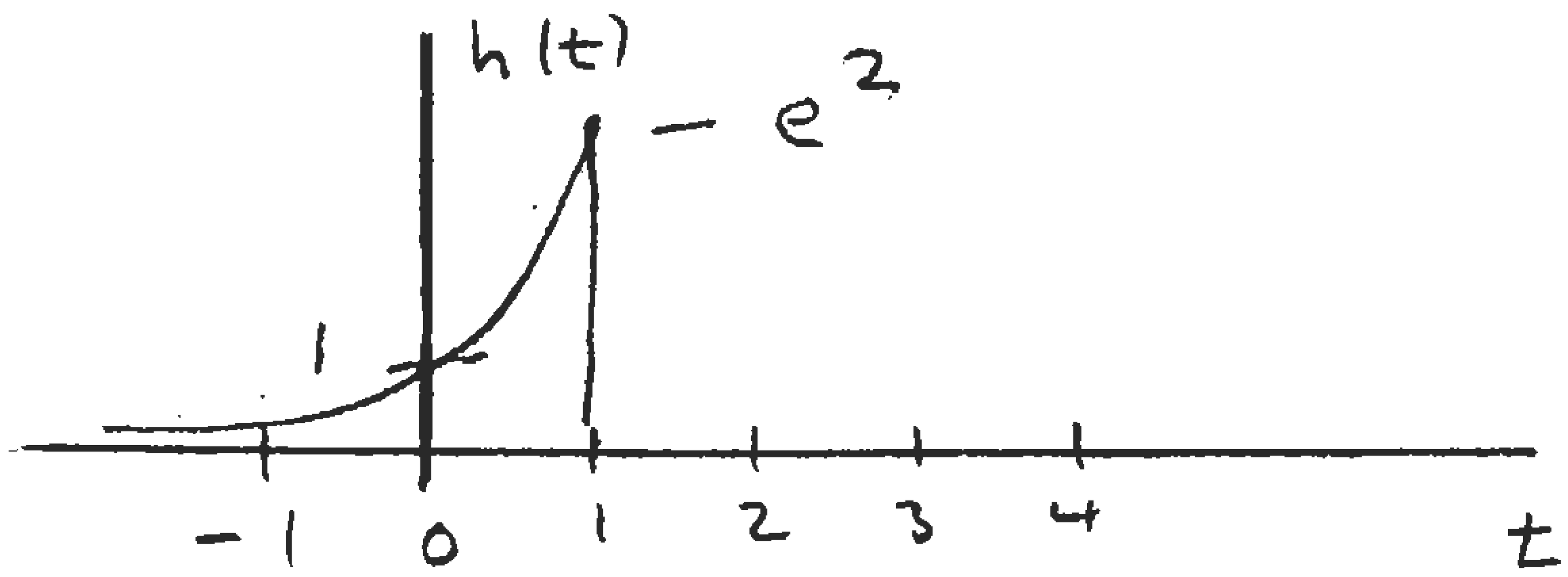
In either case, the result will look generally like



(6) $x(t) = \sigma(t) - 2\sigma(t-2) + \sigma(t-5)$

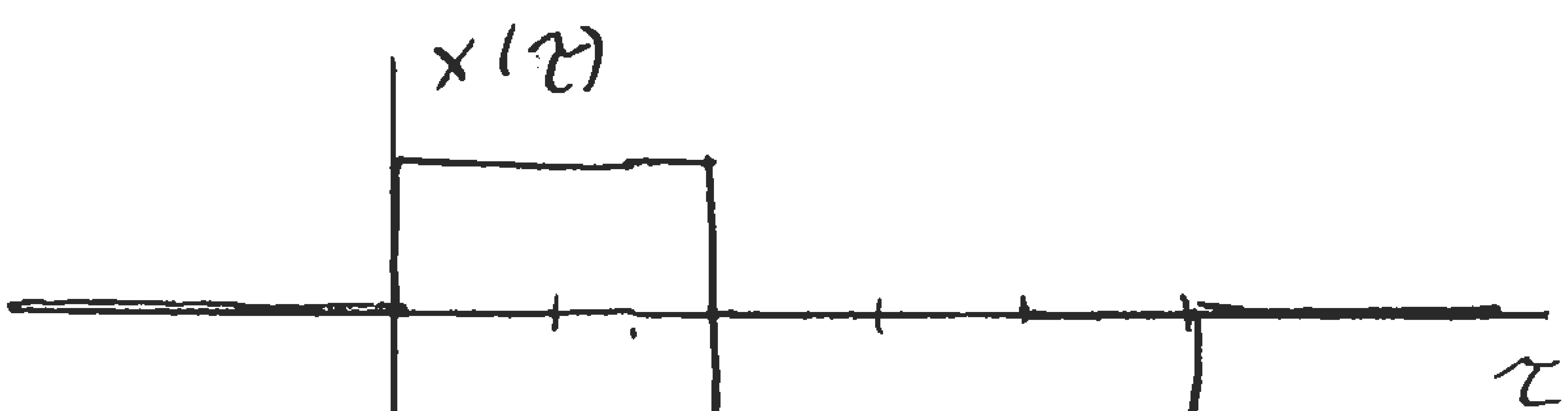


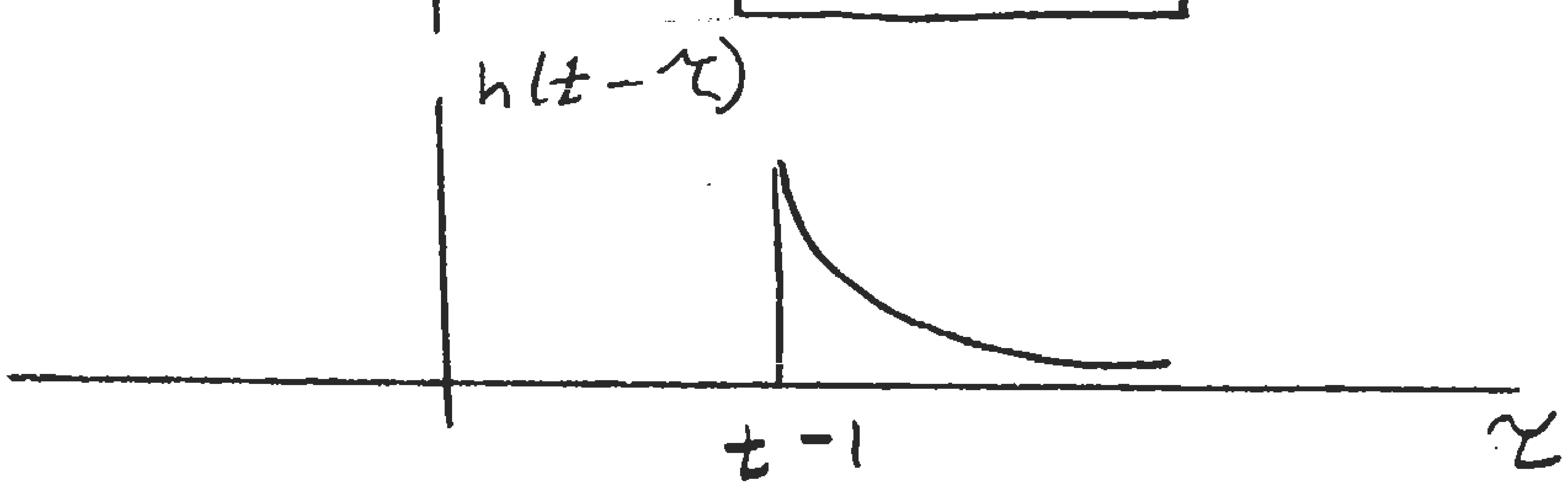
$h(t) = e^{2t}\,\sigma(1-t)$



$y(t) = h(t) * x(t)$

Use flip & slide to get feel for answer:





Depending on the value of t, there are 4 cases:

$t > 6$: For this case, there is no overlap, so

$$y(t) = 0, \qquad t > 6$$

$3 < t < 6$: For this case,

$$y(t) = \int_{t-1}^{5} e^{2(t-\tau)} (-1) \, d\tau$$

$$= e^{2t} \int_{t-1}^{5} (-1) e^{-2\tau} \, d\tau$$

$$= e^{2t} \frac{1}{2} e^{-2\tau} \Big|_{\tau = t-1}^{5}$$

$$= e^{2t} \cdot \frac{1}{2} \left[e^{-10} - e^{-2(t-1)} \right]$$

$1 < t < 3$: For this case,

$$y(t) = \int_{t-1}^{2} e^{2(t-\tau)} (1) \, d\tau + \int_{2}^{5} e^{2(t-\tau)} (-1) \, d\tau$$

$$= -\frac{1}{2} e^{2t} e^{-2\tau} \Big|_{\tau = t-1}^{2} + \frac{1}{2} e^{2t} e^{-2\tau} \Big|_{\tau = 2}^{5}$$

$$= \frac{1}{2} e^{2t} \left(e^{-2(t-1)} - e^{-4} \right) + \frac{1}{2} e^{2t} \left(e^{-10} - e^{-4} \right)$$

$t < 1$: For this case,

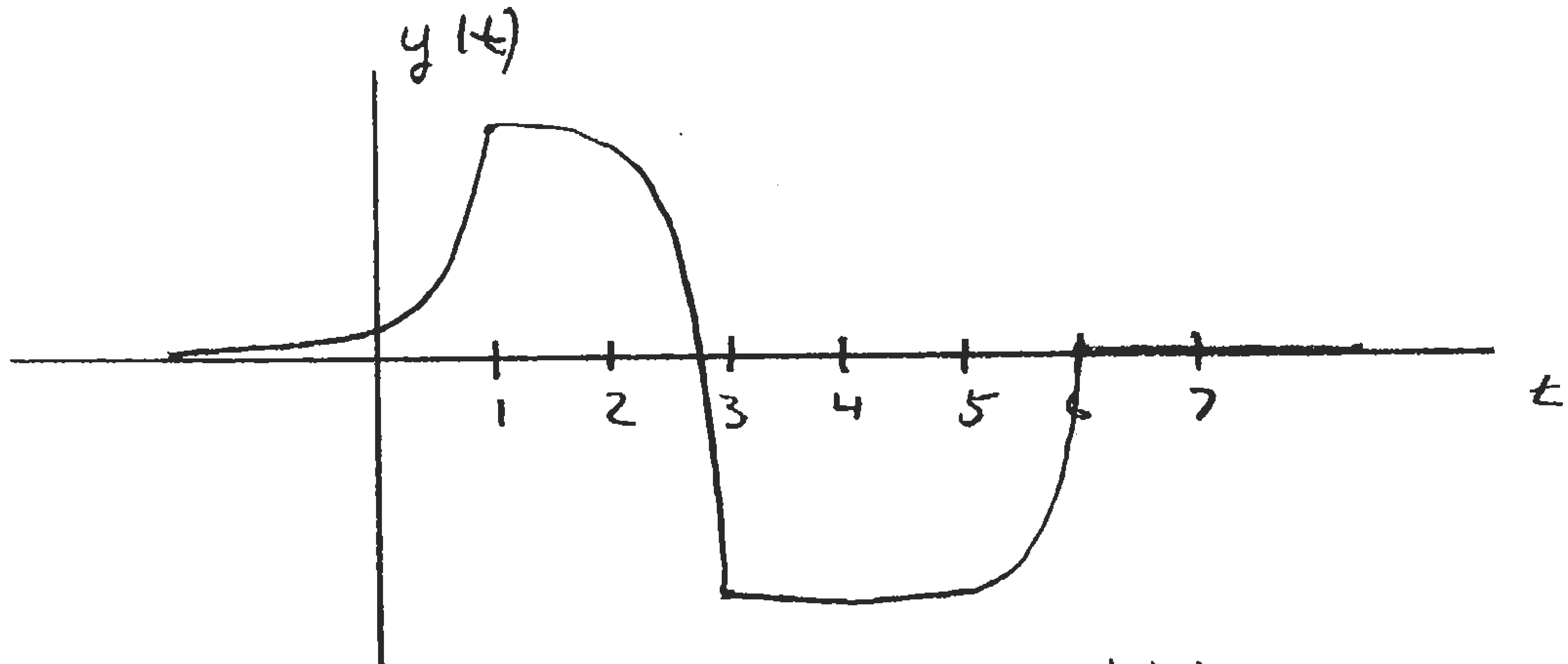
$$y(t) = \int_{0}^{2} e^{2(t-\tau)} (1) \, d\tau + \int_{2}^{5} e^{2(t-\tau)} (-1) \, d\tau$$

$$= \frac{1}{2}e^{2t}\left(e^{0} - e^{-4}\right) + \frac{1}{2}e^{2t}\left(e^{-10} - e^{-4}\right)$$

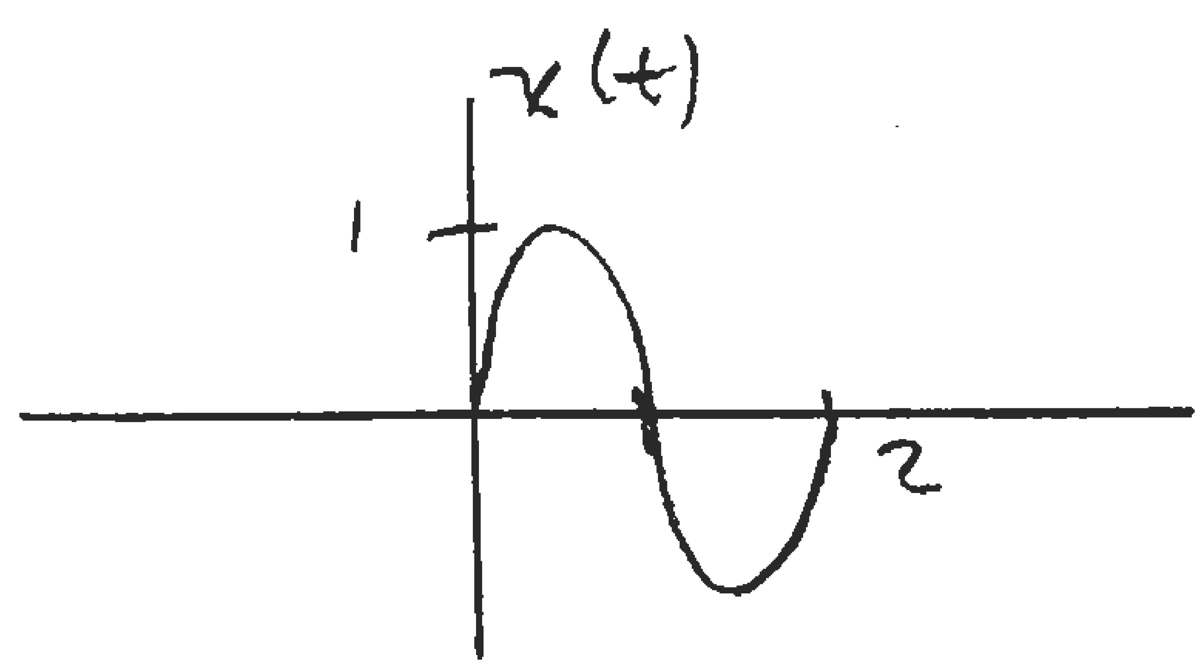
Simplifying,

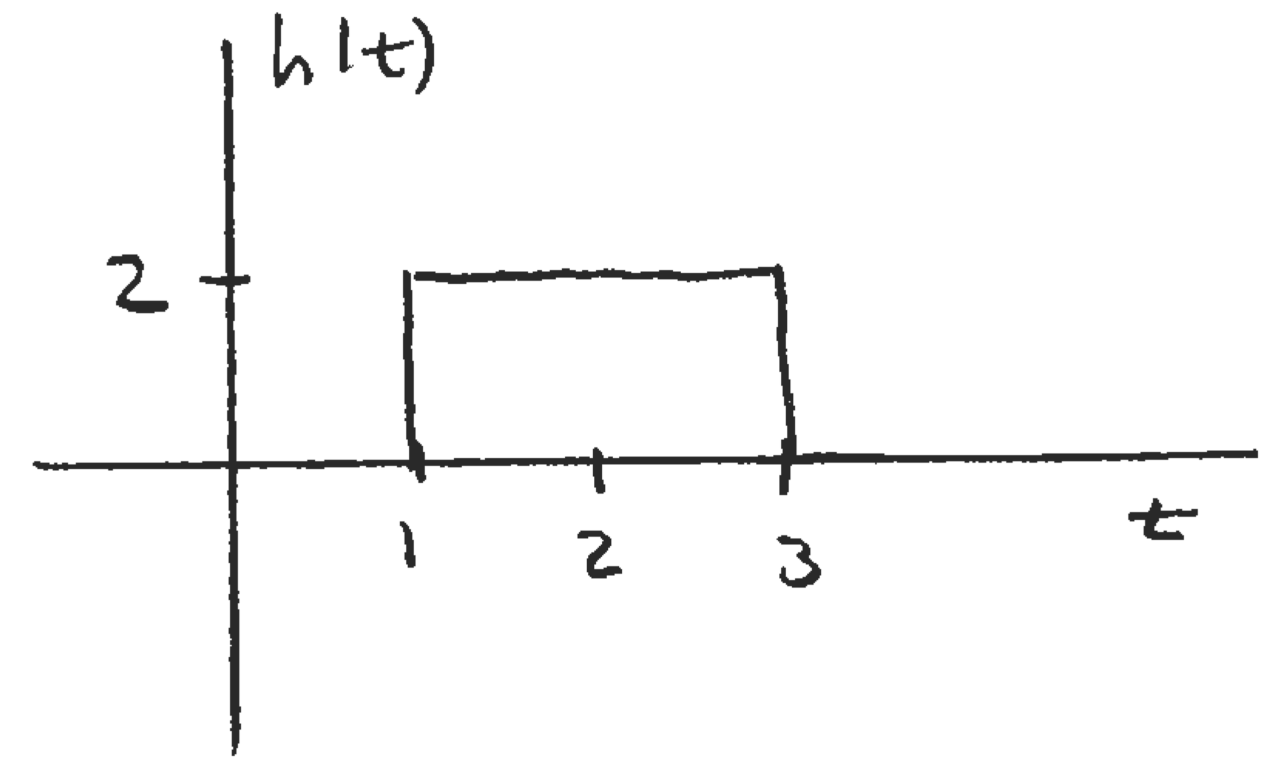
$$y(t) = \begin{cases} \frac{1}{2}e^{2t}\left(1 - 2e^{-4} + e^{-10}\right), & t < 1 \\ \frac{1}{2}e^{2} + \frac{1}{2}e^{2t}\left(e^{-10} - 2e^{-4}\right), & 1 < t < 3 \\ \frac{1}{2}e^{2t-10} - \frac{1}{2}e^{2}, & 3 < t < 6 \\ 0, & t > 6 \end{cases}$$

Sketch of $y(t)$:

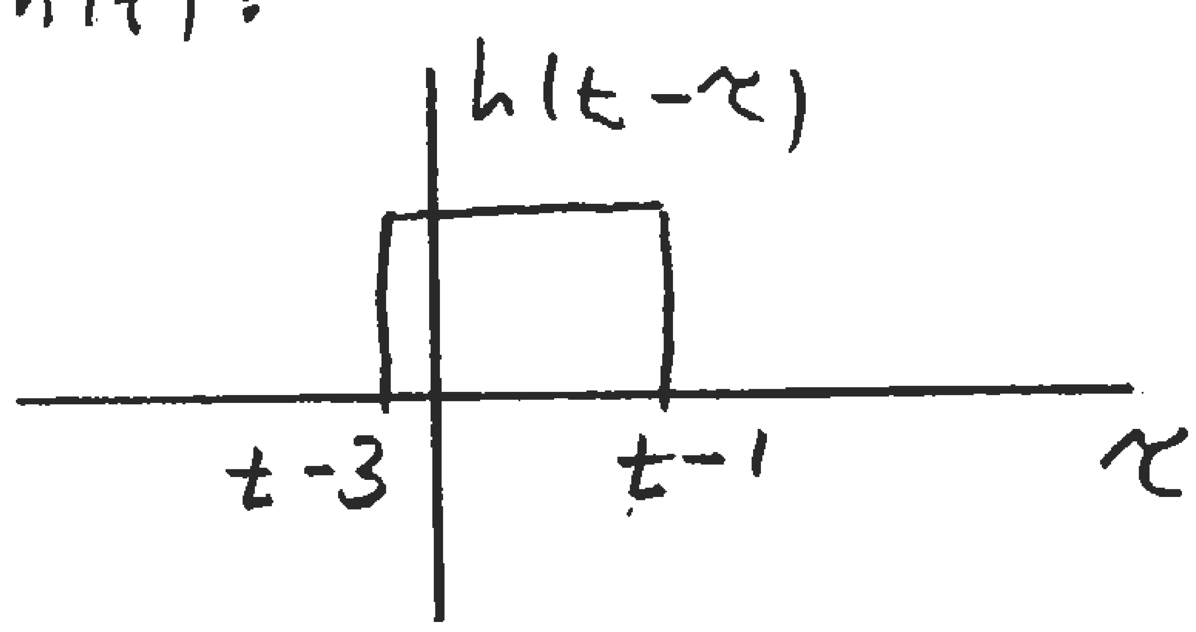


(c)





Flip $h(t)$:



There are 4 cases:

$t<1$: In this case, there is no overlap, so

$$y(t)=0$$

$1<t<3$: $$y(t)=\int_0^{t-1} 2\cdot \sin \pi\tau \, d\tau$$

$$= \frac{-2}{\pi} \cos \pi\tau \Big|_{\tau=0}^{t-1}$$

$$= \frac{-2}{\pi}\left[\cos \pi(t-1) - 1\right]$$

$$= \frac{2}{\pi}\left[1+\cos \pi t\right]$$

$3<t<5$: $$y(t)=\int_{t-3}^{2} 2 \sin \pi\tau \, d\tau$$

$$= -\frac{2}{\pi} \cos \pi\tau \Big|_{\tau=t-3}^{2}$$

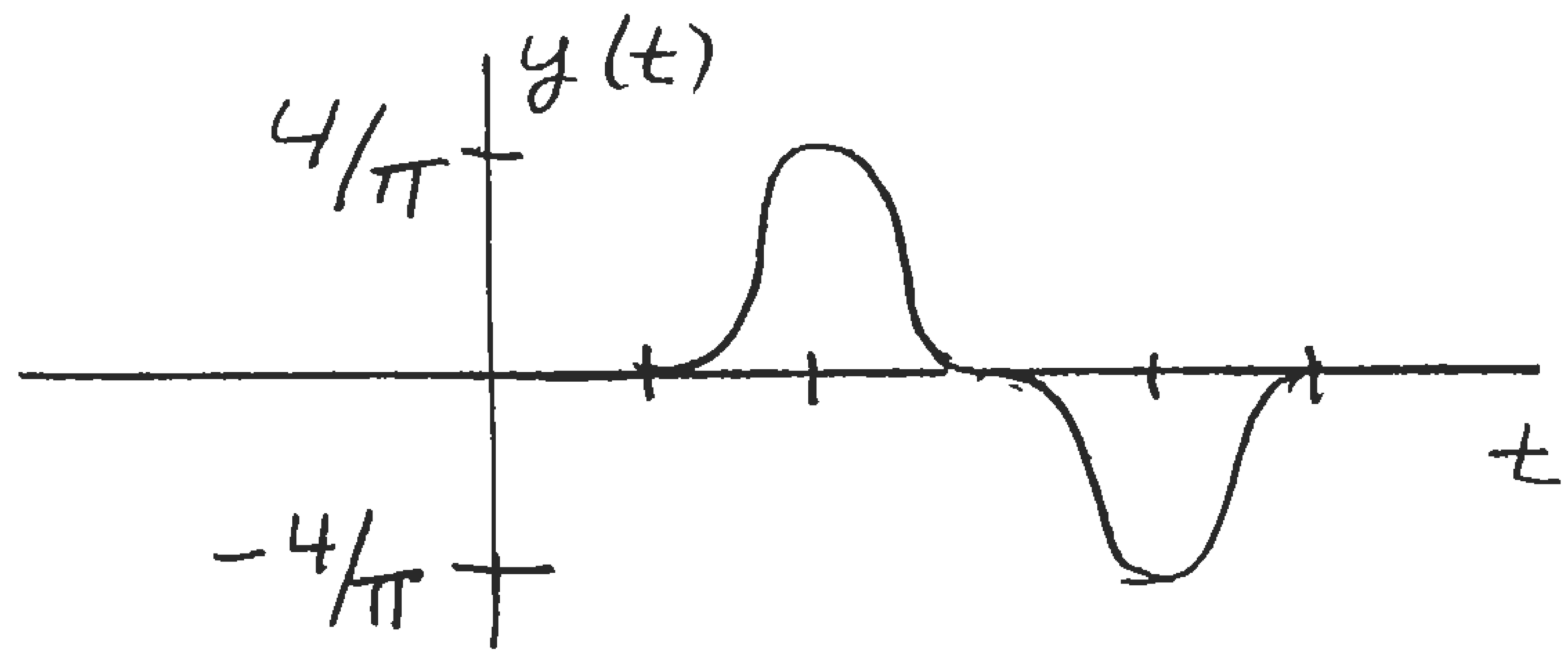
$$= -\frac{2}{\pi}\left[\cos \pi 2 - \cos \pi(t-3)\right]$$

$$= -\frac{2}{\pi}\left[1+\cos \pi t\right]$$

$t>5$: There is no overlap, so $y(t)=0$.

Therefore,

$$y(t)=\begin{cases} \frac{2}{\pi}(1+\cos \pi t) & 1<t<3 \\ -\frac{2}{\pi}(1-\cos \pi t) & 3<t<5 \\ 0 & \text{else} \end{cases}$$



(d) $y(t) = h(t) * x(t)$ $\qquad$ $x(t) = a + bt$

$\quad = x(t) * h(t)$ $\qquad$ $h(t) = \frac{4}{3}\left[r(t) - r(t-1)\right]$

$\qquad\qquad\qquad\qquad\qquad - \frac{1}{3}\delta(t-2)$

$$y(t) = \int_0^1 \frac{4}{3}\left[a + b(t-\tau)\right] d\tau$$

$$+ \int -\frac{1}{3}\delta(\tau-2)\left[a + b(t-\tau)\right] dt$$

$$= \frac{4}{3}\left[a + bt\right]\tau \Big|_{\tau=0}^{1} - \frac{2}{3} b\tau^2 \Big|_{\tau=0}^{1}$$

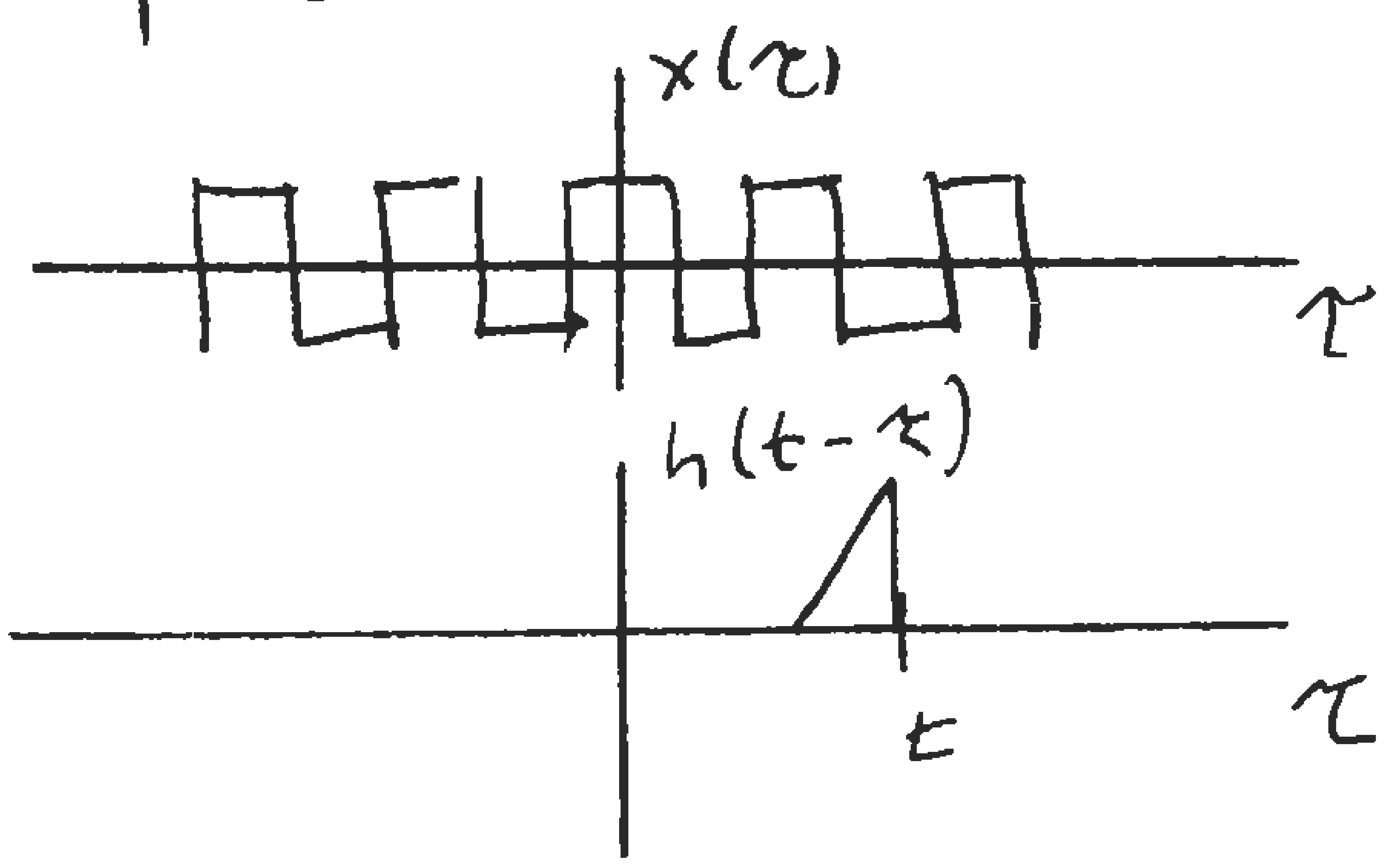
$$- \frac{1}{3}\left[a + b(t-2)\right]$$

$$= \frac{4}{3}\left[a + bt\right] - \frac{2}{3}b - \frac{1}{3}a + \frac{2}{3}b - bt]$$

$$= a + bt$$

So $y(t) = x(t)$!!!

(e) Flip & Slide:



when $h(t)$ overlaps a positive pulse,

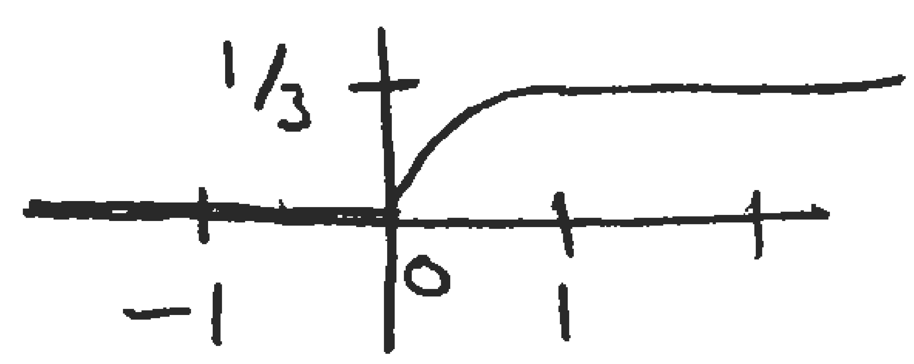
$$y(t) = \int 1 \cdot h(t-\tau)\, d\tau = 1/3$$

when $h(t)$ overlaps a negative pulse

$$y(t) = -1/3$$

If $h(t)$ is convolved with a step, $\sigma(t)$, the result is

$$h(t) * \sigma(t) =$$



Therefore, $h(t) * x(t)$ should look like