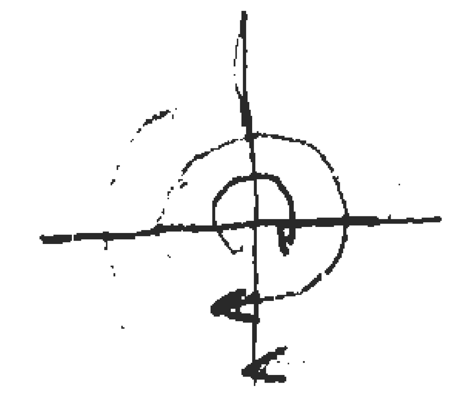
UE Fluids | F17 Solution | Fall '03

$u_1 = \frac{y}{x^2+y^2} \qquad v_1 = \frac{-x}{x^2+y^2} \qquad$ "$\frac{1}{r}$ vortex"



$u_2 = V_\infty \qquad v_2 = 0 \qquad$ uniform flow $\longrightarrow$

$$\boxed{u_3 = u_1 + u_2 = \frac{y}{x^2+y^2} + V_\infty} \quad \boxed{v_3 = v_1 + v_2 = \frac{-x}{x^2+y^2}}$$

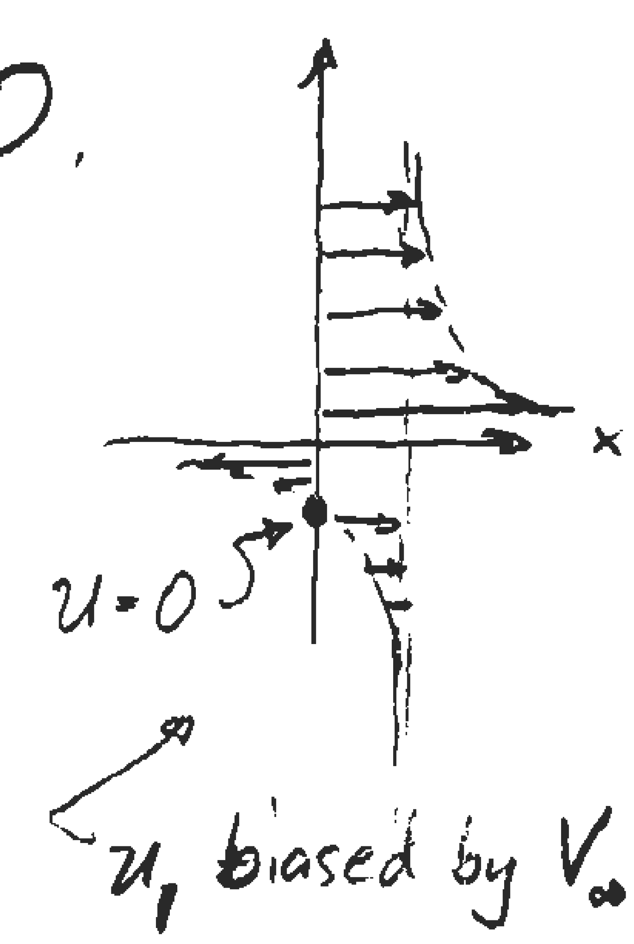
$$p_3 = p_0 - \frac{1}{2}\rho\left(u_3^2 + v_3^2\right) = p_0 - \frac{1}{2}\rho\left[\frac{y^2}{(x^2+y^2)^2} + \frac{2yV_\infty}{x^2+y^2} + V_\infty^2 + \frac{x^2}{(x^2+y^2)^2}\right]$$

$$\boxed{p_3 = p_0 - \frac{1}{2}\rho\left[\frac{1+2yV_\infty}{x^2+y^2} + V_\infty^2\right]}$$

Maximum pressure is where $u_3^2 + v_3^2$ is minimum.

Note that on y-axis where $x=0$, we have $v_3 = 0$.
Also, at $y = -1/V_\infty$ we also have $u_3 = 0$

$$\Rightarrow \text{max } p_3 \text{ at } x, y = 0, \frac{-1}{V_\infty}$$



Alternative mathematical approach (hard way)

set $\frac{\partial p_3}{\partial x} = 0$ and $\frac{\partial p_3}{\partial y} = 0$

$$\frac{\partial p_3}{\partial x} = -\frac{1}{2}\rho\frac{1+2yV_\infty}{(x^2+y^2)^2}(-2x) = 0 \Rightarrow (1+2yV_\infty)x = 0 \qquad (1)$$

$$\frac{\partial p_3}{\partial y} = -\frac{1}{2}\rho\frac{1+2yV_\infty}{(x^2+y^2)^2}(-2y) - \frac{1}{2}\rho\frac{2V_\infty}{x^2+y^2} = 0 \Rightarrow (1+2yV_\infty)y - (x^2+y^2)V_\infty = 0$$

or $(1+yV_\infty)y - x^2V_\infty = 0 \quad (2)$

Two possibilities from equation (1)

a) $1+2yV_\infty = 0$, $x \neq 0 \rightarrow y = \frac{-1}{2V_\infty}$

Plug into equation (2) $\rightarrow \frac{1}{2}\cdot\left(\frac{-1}{2V}\right) - x^2V_\infty = 0 \rightarrow x^2 = \frac{-1}{4V_\infty}$ no real solution

b) $1+2yV_\infty \neq 0$, $\boxed{x=0}$

Plug into equation (2) $\rightarrow (1+yV_\infty)y = 0$

$\nearrow$ $y \stackrel{?}{=} 0$ Nope. Inconsistent with (1).

$\searrow$ $1+yV_\infty = 0 \rightarrow \boxed{y = \frac{-1}{V_\infty}}$ ✓