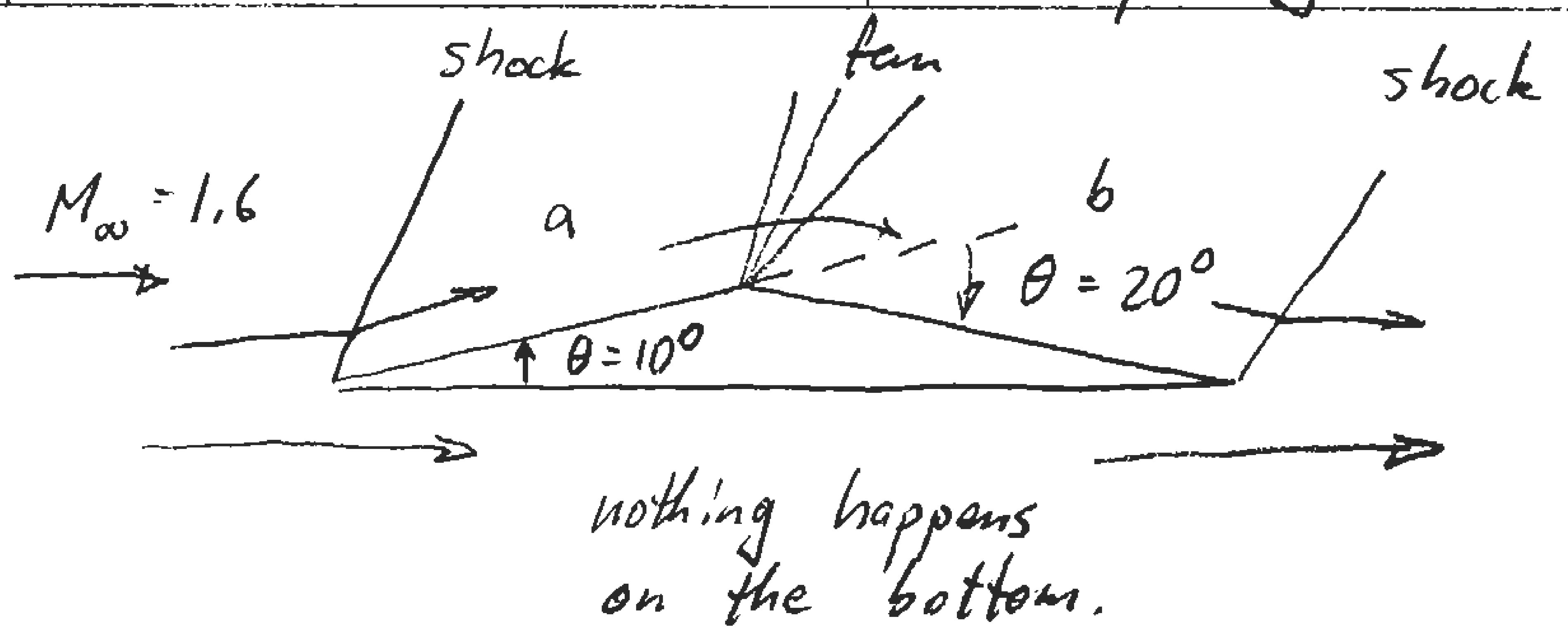
UE Fluids | F17 + F18 Solution | Spring '04

Flow sketch



a) Oblique Shock. $M_1 = 1.6,\ \theta = 10^\circ \rightarrow \beta = 51^\circ$ (chart, Anderson p. 513)

$$M_{n_1} = M_1 \sin 51^\circ = 1.6 \cdot 0.777 = 1.243$$

$$p_2/p_1 = f(M_{n_1}) = 1.636$$ (Anderson eq. 9.16, or Appendix B table.)

Since $p_1 = p_\infty \rightarrow$ $\boxed{p_2 = 1.636\, p_\infty = p_a}$

$$M_{n_2} = f(M_{n_1}) = 0.817 \quad \text{(eq. 9.14)}$$

$$\boxed{M_2 = M_a = \frac{M_{n_2}}{\sin(51^\circ - 10^\circ)} = 1.245}$$

b) Expansion fan. $M_1 = M_a = 1.245$, $\nu(M_1) = 4.7^\circ$

$$\nu(M_2) = \nu(M_1) + 20^\circ = 24.7^\circ \longrightarrow M_2 = 1.94$$

$$p_2 = p_{o_2}\left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\frac{-\gamma}{\gamma-1}}$$

$p_{o_2} = p_{o_1} = p_{o_a}$ (behind oblique shock).

$$p_{o_a}/p_{o_\infty} = f(M_{n_1}) = 0.987$$

$$p_{o_a} = 0.987\, p_{o_\infty} = 0.987\, p_\infty \left[1 + \frac{\gamma-1}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} = 4.195\, p_\infty$$

$$\therefore\ p_2 = 4.195\, p_\infty \left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\frac{-\gamma}{\gamma-1}} = \boxed{0.588\, p_\infty = p_b}$$

Note: If we neglect the oblique shock's loss: $p_{o_b} = p_{o_a} = p_{o_\infty} \rightarrow p_2 = 0.596\, p_\infty$ (not quite correct)

c) $$L' = L'_a + L'_b = (p_\infty - p_a)\frac{c}{2} + (p_\infty - p_b)\frac{c}{2} = (1 - 1.636)\, p_\infty \frac{c}{2} + (1 - 0.588)\, p_\infty \frac{c}{2}$$

$$\boxed{L' = -0.112\, p_\infty c}$$

$$D' = D'_a + D'_b = -L'_a \tan 10^\circ + L'_b \tan 10^\circ = \left[-(1 - 1.636) + (1 - 0.588)\right] \tan 10^\circ\, p_\infty \frac{c}{2}$$

$$\boxed{D' = 0.0924\, p_\infty c}$$

Using $\frac{1}{2}\rho_\infty V_\infty^2 = \frac{\gamma}{2} p_\infty M_\infty^2 = 1.792\, p_\infty \rightarrow$

$$\boxed{\begin{aligned} c_l &= L' / \tfrac{1}{2}\rho_\infty V_\infty^2 = -0.0625 \\ c_d &= D' / \tfrac{1}{2}\rho_\infty V_\infty^2 = 0.0516 \end{aligned}}$$