C13

1. Solve the following recurrence equation using the iteration method. Show all the steps in your derivation.

$$T(n) = \begin{cases} c & n = 1 \\ aT\left(\frac{n}{b}\right) + cn & n > 1 \end{cases}$$

Substitute the value of T(n) from the recurrence equation:

aT(n/b) + cn

$\Rightarrow$ a(aT((n/b)/b) + c(n/b)) + cn

$\Rightarrow$ $a^2T(n/b^2) + cn(a/b) + cn$

$\Rightarrow$ $a^2T(n/b^2) + cn((a/b) + 1)$

$\Rightarrow$ $a^2(aT((n/b^2)/b) + cn/b^2) + cn((a/b) + 1)$

$\Rightarrow$ $a^3T(n/b^3) + cn(a^2/b^2) + cn((a/b) + 1)$

$\Rightarrow$ $a^3T(n/b^3) + cn((a^2/b^2) + (a/b) + 1)$

…

$\Rightarrow$ $a^kT(n/b^k) + cn((a^{k-1}/b^{k-1}) + (a^{k-2}/b^{k-2}) + \ldots + (a^2/b^2) + (a/b) + 1)$

When $k = \log_b n$,

$\Rightarrow n = b^k$

T(n) $= a^kT(1) + cn(a^{k-1}/b^{k-1} + ... + a^2/b^2 + a/b + 1)$

$= a^kc + cn(a^{k-1}/b^{k-1} + ... + a^2/b^2 + a/b + 1)$

$= ca^k + cn(a^{k-1}/b^{k-1} + ... + a^2/b^2 + a/b + 1)$

$= cna^k/b^k + cn(a^{k-1}/b^{k-1} + ... + a^2/b^2 + a/b + 1)$

$= cn(a^k/b^k + ... + a^2/b^2 + a/b + 1)$