Basic Mechanics notes

Parallel tutorial session # 1 : BASIC MECHANICS

08 / 07 / 2006

Welcome and introduction to GEM^4

Subra Suresh, Roger Kamm

- international, interdisciplinary, inter-institutional initiative started early 2006
  nucleus : MIT, NSU, Harvard, Institut Pasteur
  engineering, biology, medicine & public health intersected
  paradigm for global cooperation through research, training & translation
  networking opportunities
  infrastructure capabilities
- training program : summer school (2007 in Singapore, focus on cancer)*
  also offered by GEM^4: distinguished lecture series
  junior scientist program
  * with the newly announced SMART : Singapore-MIT Alliance for Research and Technology Center in Singapore
  GEM^4 co-sponsors the next world-wide meeting on biomechanics in 2010.

- Cell and molecular mechanics in biomedicine, with a focus on infectious diseases.
  from a course developed at MIT by Profs. Patrick Doyle, Alan Grodzinsky & R. Kamm.
  lab info : http://www.openwetware.org/wiki/GEM4labs
  breakdown of participants : 30% life scientists, 70% engineers
  poster sessions to share research knowledge.
  research proposals to be developed during the summer schools, presented in 3-4 pages
  direct questions to Maggie Sullivan or Roger Kamm : sullmag@ rdkamm@ mit.edu

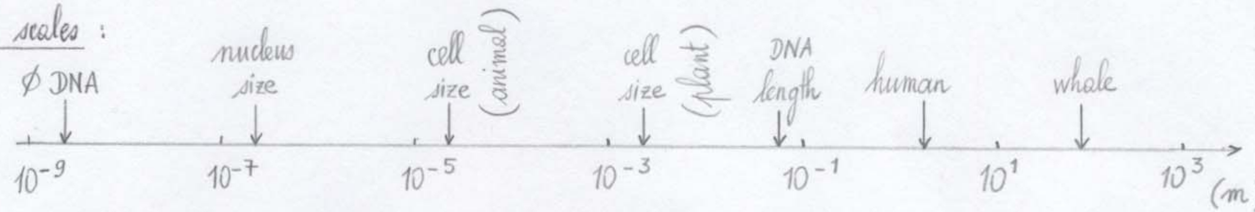
Simple statistical mechanics for biological systems

L. Mahadevan

- Questions : What is the goal ?
  starting with the central dogma in biology (Crick) : DNA → RNA → proteins → organelles → cells → tissue → organism → ecosystem
  (and feedback !), essentially increasing length scale

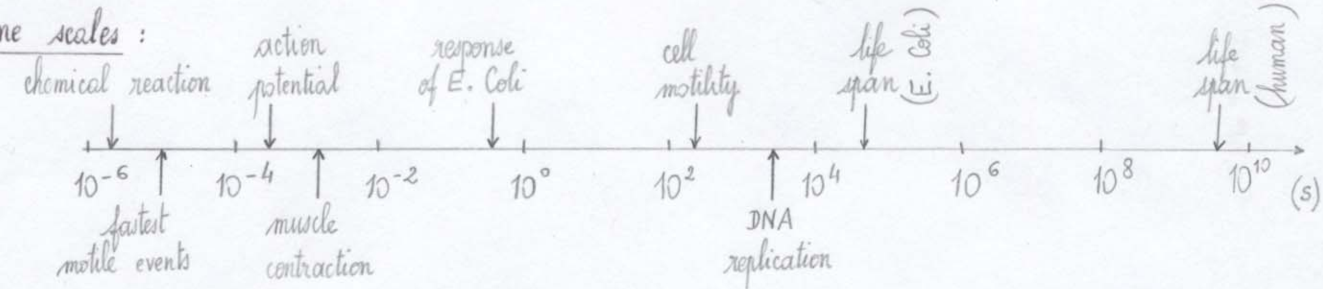
Hence, length and time scales and "out of equilibrium" principles matter.

Length scales :



cells are the fundamental units of life : smallest to function independently
cell size $\propto$ amount of DNA enclosed.

Time scales :



Out of equilibrium biology : soft , wet , dynamic , warm
information (encoded in DNA, ...) , energy, matter
we wish to couple energy and matter : biology is warm, hence in motion, hence energy.
(dynamic & directed)

Energy scales : thermal energy used as a ruler

energy / mole / °K $\sim$ RT
(of ideal gas) $= \frac{1}{2}$ RT per degree of freedom
(3 d.o.f. for ideal gas)
$= 8.3$ J / mole / °K for ideal gas

energy / molecule $\sim RT / \mathcal{N}_A = k_B T$
(and per degree of freedom)

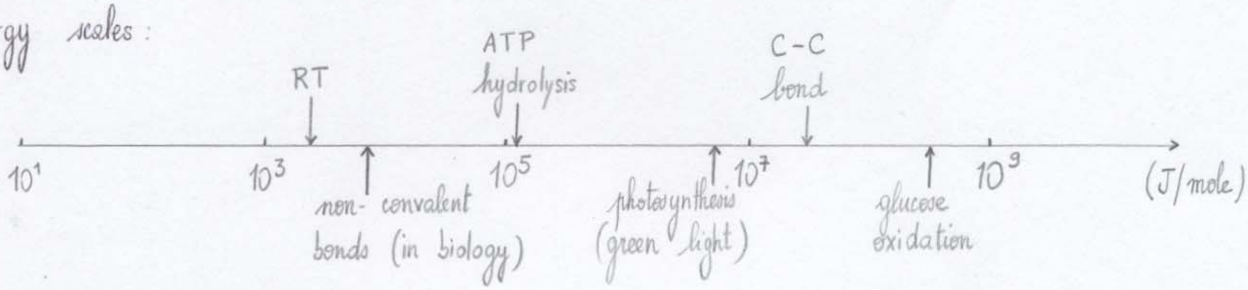
with Avogadro's number $\mathcal{N}_A \sim 6 \times 10^{23}$ and Boltzmann's constant $k_B =$
1.38×10^{-23} J/ molecule / °K

$$\mathcal{U}_{molecule} \sim 4 \times 10^{-21} \text{ J}$$
$$\sim 4 \times 10^{-12} \text{ N} \times 10^{-9} \text{ m}$$

hence force scale of pN and length scale of nm. (pico Newtons , nanometers)

Coupled interactions : chemical (physical)
electrical
mechanical
} grand goal is to understand how these interact (are organized) in space and time

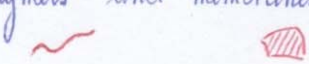
Energy scales :



Over the past decade, much progress has been made experimentally and technically.
Down on small scales, biology is geometrically dominated by filaments and membranes
which is connected with chemical malleability
which makes for physical complexity
(nonlinearity)

Outline :

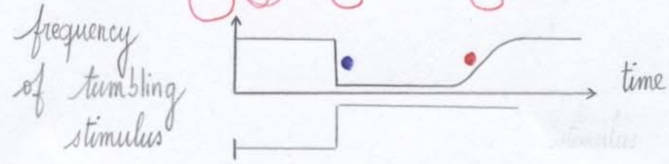
- random walks & diffusion
- drag, mobility, Boltzmann's law, Stokes - Einstein
- biological forces & energies
- physics, mechanics and mechano-chemistry of polymers and membranes



Book reference : E. Coli in motion (2005) by H. C. Berg

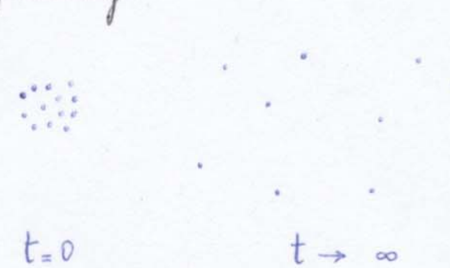
- a bacteria tumbles randomly in a homogeneous environment, then more directedly in the presence of a chemoattractant (response to a stimulus) ; when the chemoattractant diffuses away, randomness reappears (adaptation)

sensing and movement are coupled :



- in the absence of any active processes,
what happens to a bolus of chemoattractant ?
Carnot, Helmholtz, Boltzmann, Gibbs : maximize the disorder
statistical probability

random walks (in 1-D) } and diffusion }
microscopic } macroscopic }



- from kinetic energy: $\frac{1}{2} k_B T = \frac{1}{2} m \langle v^2 \rangle$

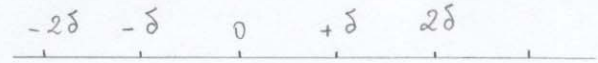
$\langle v^2 \rangle = k_B T / m$ mean squared velocity

for a lysozyme $m \sim 14$ kg / mole and 6×10^{23} molecules / mole

$\sqrt{\langle v^2 \rangle} \sim 14$ m / s very high !

but collisions and dissipation $\Rightarrow$ no net motion of the lysozyme

velocity v, step size $\delta = \pm v \tau$ with τ: time between collisions.



① probability of going in either direction $p = 1/2$

② each step independent of others

- with N particles at origin at $t = 0$

$$x_i(n) = x_i(n-1) \pm \delta \quad \begin{cases} i: & \text{particle label} \\ n: & \text{number of steps} = t/\tau \end{cases}$$

$$\langle x(n) \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i(n) = \langle x_i(n-1) \rangle = \dots = \langle x_i(0) \rangle = 0$$

$\langle x(n) \rangle = 0$ mean location of particles

- spread of distribution : variance

$$x_i^2(n) = x_i^2(n-1) + \delta^2 \pm 2\delta x_i(n-1)$$

$$\langle x_i^2(n) \rangle = \langle x_i^2(n-1) \rangle + \delta^2 = n \delta^2$$

$$\langle x^2(t/\tau) \rangle = \frac{1}{2} \cdot 2\delta^2/\tau \cdot t = 2 \mathcal{D} t \quad \text{with } \mathcal{D} = \frac{1}{2}\frac{\delta^2}{\tau}$$

mean square proportional to time

$\sqrt{\langle x^2(t/\tau) \rangle} \propto t^{1/2}$ slow (diffusion)

$\mathcal{D} \sim \sqrt{\langle v^2 \rangle}\, \delta \sim 10$ m/s . 10^{-10} m $\sim 10^{-9}$ m²/s or 10^{-5} cm²/s

$t \big|_{10^{-4}\,\text{cm} \sim 1\,\mu\text{m}} \sim (10^{-6}\text{ m}) \div (10^{-9}\text{ m}^2/\text{s}) \sim 10^{-3}$ s quick on small scale

$t \big|_{1\,\mu\text{m}} \sim 10^4$ s or 10 hours slow on large scale

Foundations of continuum mechanics

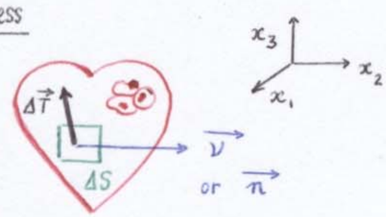
Geert Schmid-Schönbein

Newton's law $\vec{F} = m \frac{d\vec{v}}{dt}$ was voted to be the most important human discovery.
is at the core of biology (force & velocity related).
A major part of mechanics today is to understand and measure forces.

Forces — what should the mass m be in biology? neighboring cells influence one cell
Newton's law is not as useful in this discrete form as in engineering

force: interaction between discreet identifyable objects
but another definition of force / interaction is needed in biology

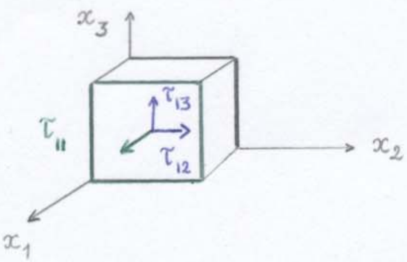
Stress



$\left.\begin{array}{ll} \Delta S & \text{surface area} \\ \vec{\nu} & \text{outer normal} \\ \Delta\vec{T} & \text{force exerted} \end{array}\right\} \quad \lim_{\Delta S \to 0} \frac{\Delta T}{\Delta S} = \overset{\nu}{\vec{T}}$

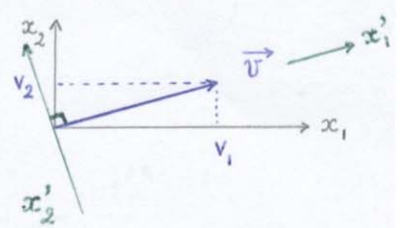
and with
$\vec{\nu} \parallel x_1$ axis : $\overset{\nu}{\vec{T}}{}^{(1)}$: τ_{11} τ_{12} τ_{13}
$\vec{\nu} \parallel x_2$ axis : $\overset{\nu}{\vec{T}}{}^{(2)}$: τ_{21} τ_{22} τ_{23}
$\vec{\nu} \parallel x_3$ axis : $\overset{\nu}{\vec{T}}{}^{(3)}$: τ_{31} τ_{32} τ_{33}

stress tensor τ_{ij}
force per unit area



τ_{11} τ_{22} τ_{33} normal stress
τ_{12} τ_{13} τ_{21} ... shear stress
pressure $= -\frac{1}{3}(\tau_{11} + \tau_{22} + \tau_{33})$
from outer world onto cubic element

Principal stress . Starting with an analogy with vectors
coordinates depend on set of axes



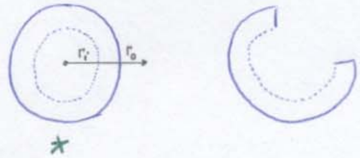
in principal coordinates system (one and only one exists):

$$\begin{pmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_{22} & 0 \\ 0 & 0 & \tau_{33} \end{pmatrix}$$

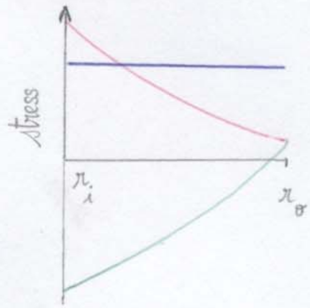
Stress deviators : the solutions to certain problems are independent of normal stresses.

$$\begin{cases} \tau'_{ij} = \tau_{ij} - \overbrace{\frac{1}{3}(\tau_{11} + \tau_{22} + \tau_{33})}^{\text{mean normal stress}} \delta_{ij} \\ \quad\;\; = \tau_{ij} + p\,\delta_{ij} \\ \delta_{ij} = 0 \text{ if } i \neq j \text{ and } 1 \text{ if } i = j \end{cases}$$

Residual stress



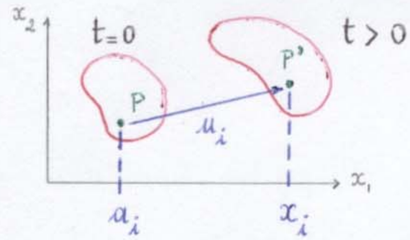
the unloaded* state is not necessarily stress-free



without
with
residual stress

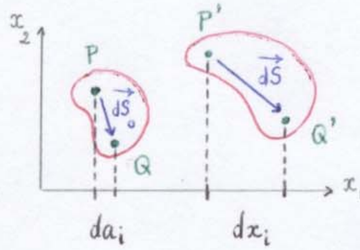
Deformation

strain
strain rate



displacement $u_i = x_i - a_i$
final - initial
includes deformation AND translation

Hooke : change in length measures deformation alone

- 

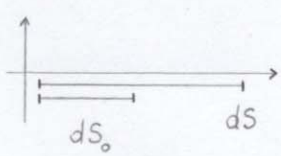
change of length $dS^2 - dS_o^2$ = strain x 2 x initial distance

<u>strain</u> E_{ij}

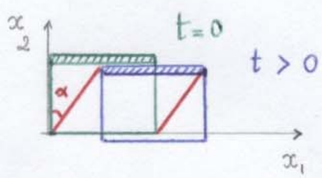
$$E_{ij} = \frac{1}{2}\left(\frac{\partial x_k}{\partial a_j} \cdot \frac{\partial x_k}{\partial a_i} - \delta_{ij}\right)$$

deformation gradients $\dfrac{\partial x_k}{\partial a_j}$

in 1-D motion



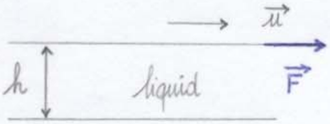
$$\begin{cases} E_{11} = \dfrac{1}{2}\left(\dfrac{dS^2}{dS_o^2} - 1\right) \\ \text{stretch ratio } \lambda = \dfrac{dS}{dS_o} \end{cases}$$



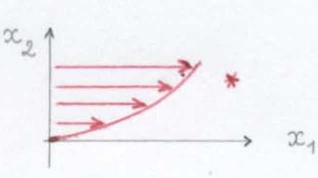
$E_{12} \approx \tan\alpha$

Reference states in biology are debatable, difficult to define (because deformable materials)

- In a fluid, what quantity is proportional to the applied stress ? strain rate



$F \sim \dfrac{u}{h}$ velocity gradient



spatial velocity gradients or <u>strain rate</u>

$$\begin{matrix} \dfrac{\partial v_1}{\partial x_1} & \dfrac{\partial v_1}{\partial x_2}* & \dfrac{\partial v_1}{\partial x_3} \\ \vdots & \vdots & \\ & & \dfrac{\partial v_3}{\partial x_3} \end{matrix}$$