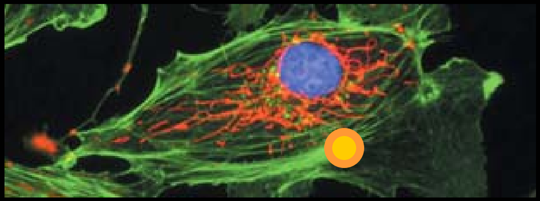
Microrheology of Complex Fluid

- ❑ Rheology: Science of the deformation & flow of matter
- ❑ Microrheology
  - Microscopic scale samples
  - Micrometer lengths



Complex shear modulus $G^*(\omega)$

$$\boxed{\sigma = G^* \varepsilon}$$

- $G^*(\omega) = G'(\omega) + j\,G''(\omega)$
- Solid *vs.* fluid
- Resistance to deformation

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Storage modulus G'

Energy storage

Elasticity ~ Solid

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Loss modulus G''

Energy dissipation

Viscosity ~ Fluid

High Frequency Microrheology Measurement

Active Method:

Magnetic microrheometer – Baush, BJ 1998

Huang, BJ 2002

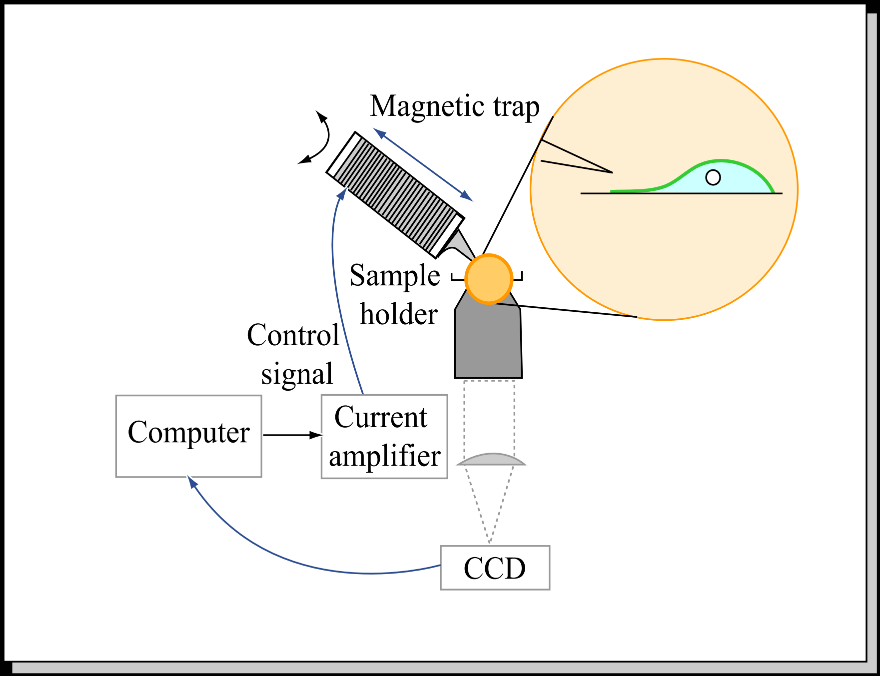
Passive Method:

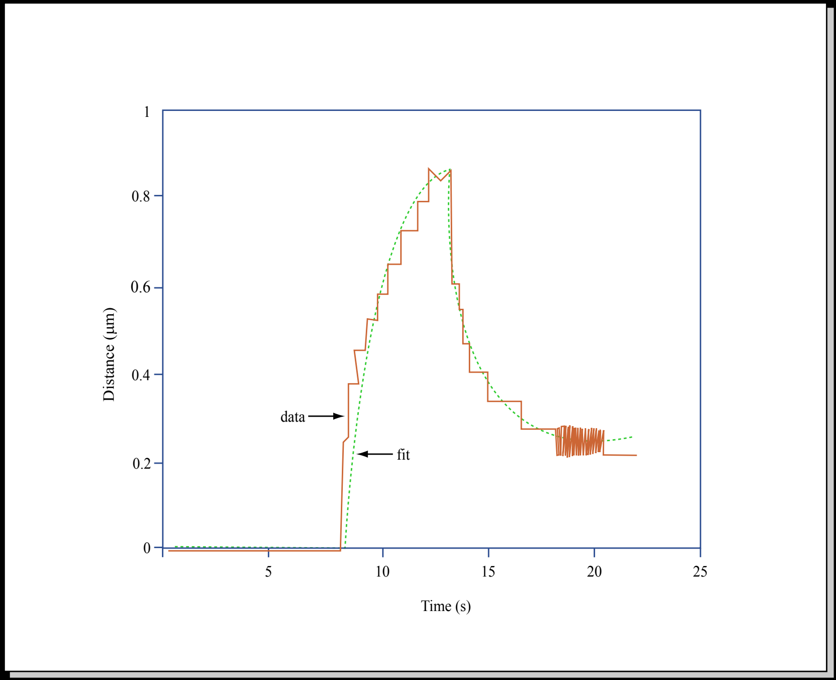
Single particle tracking – Mason, PRL 1995

Yamada, BJ 2000

Multiple particle tracking – Crocker, PRL 2000

Magnetic Microrheology





Figures by MIT OCW.

Magnetic Microrheology

5 sec Step Response

Basic Physics of Magnetic Microrheometer

Ferromagnetic particle

$$\mathbf{F} = \frac{1}{2}\mu_0 \nabla(\mathbf{m}\cdot\mathbf{H})$$

Particles cluster together!
Doesn't work!

Paramagnetic particle – no permanent magnetic moment

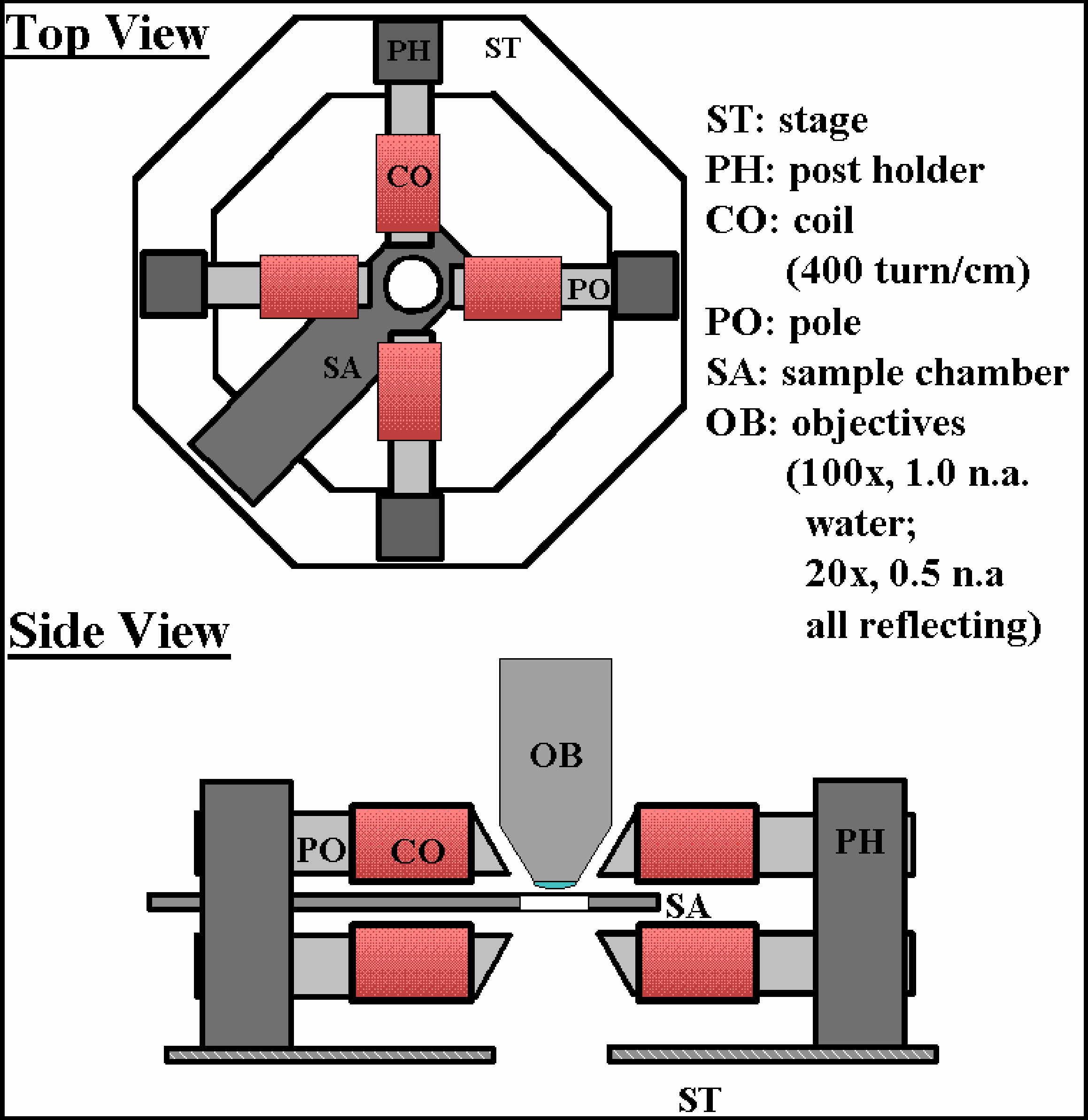
$$\mathbf{F} = \mu_0 \chi V \nabla(\mathbf{H}\cdot\mathbf{H})$$

χ is suceptibility

V is volume

Note: (1) force depends on volume of particle
(5 micron bead provide 125x more force)
(2) force depends on magnetic field GRADIENT

Magnetic manipulation in 3D



*Lower Force
nN level

*3D

*Uniform gradien

Amblad, RSI 1996
Huang, BJ 2002

Magnetic manipulation in 1D

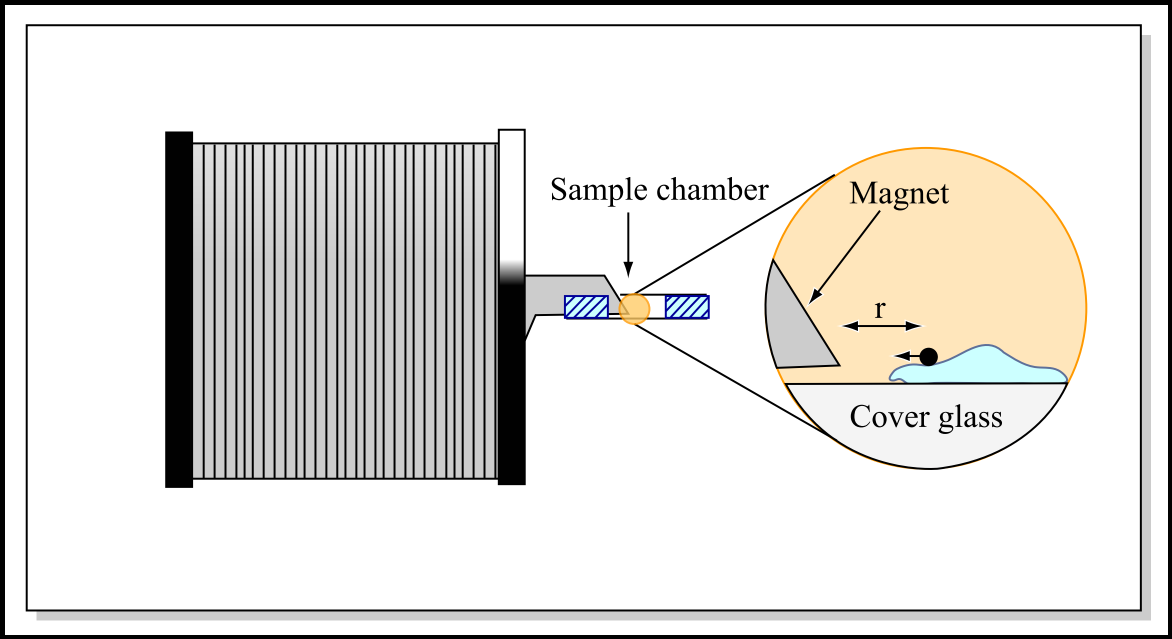


Figure by MIT OCW. After Bausch et al., 1998.

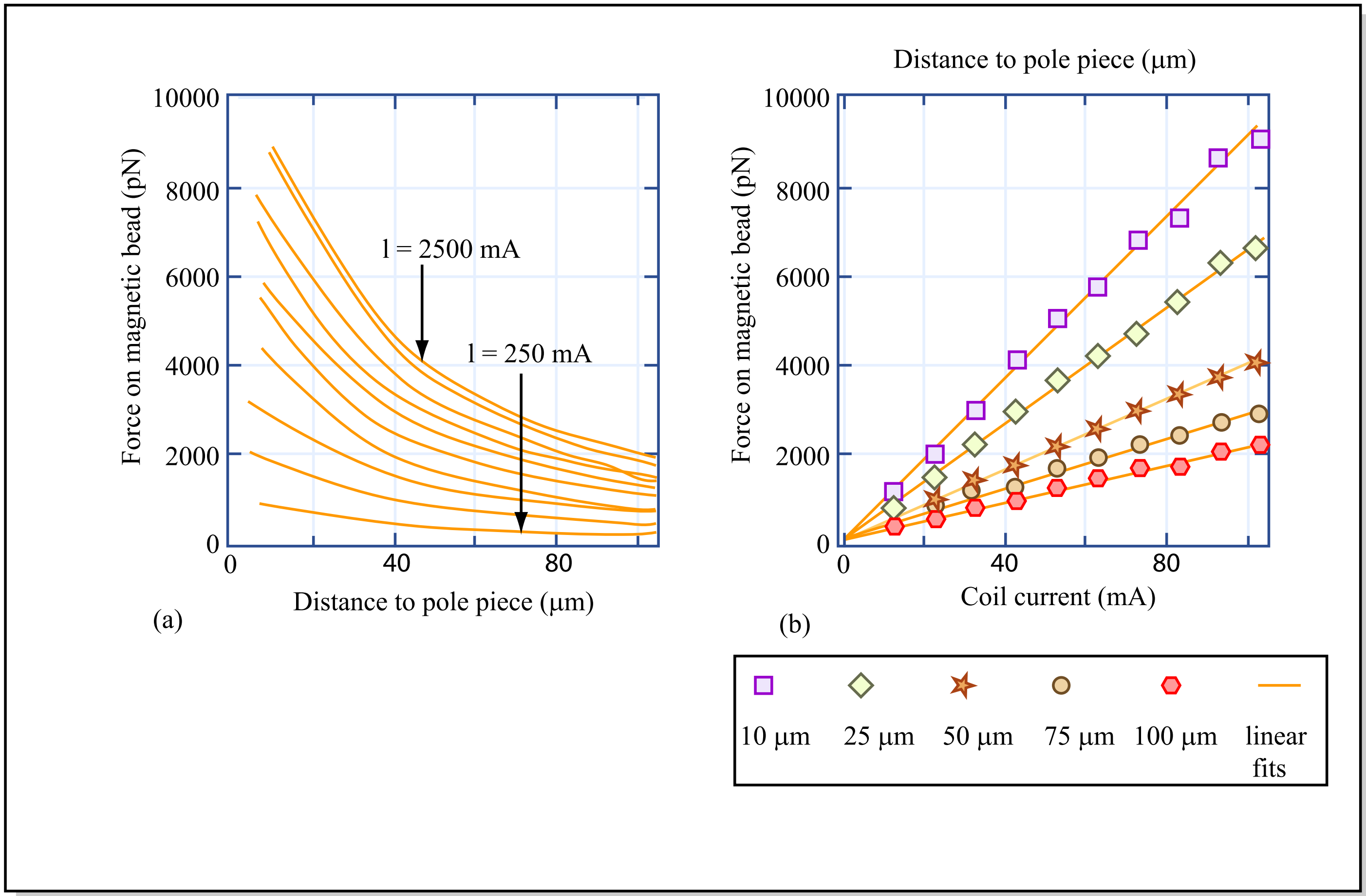
*High force
>10 nN

*Field non-uniform
Needs careful
alignment of tip
to within microns

*1D

The bandwidth of ALL magnetic microrheometer is limited by the inductance of the eletromagnet to about kiloHertz

Magnetic Rheometer Requires Calibration



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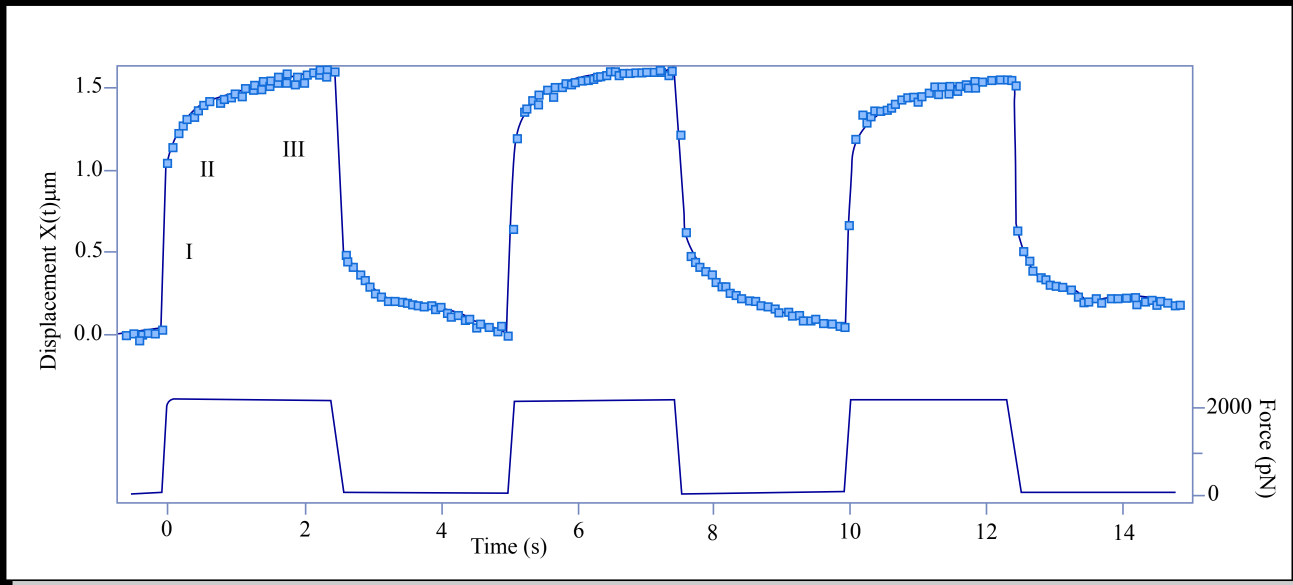
Mag Rheometer Experimental Results



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Transient responses allow fitting to micro-mechanical model

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See Fig. 5 in Bausch et al. "Local Measurements of Viscoelastic Parameters of Adherent Cell Surfaces by Magnetic Bead Microrheometry." *Biophys J* 75 (1998): 2038-2049.

Problem – Magnetic bead rolling

Solution – Injection, Endocytosis
Modeling (Karcher BJ 2003)

Model Strain Field Distribution

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See Fig. 9 in Bausch et al. "Local Measurements of Viscoelastic Parameters of Adherent Cell Surfaces by Magnetic Bead Microrheometry." *Biophys J* 75 (1998): 2038-2049.

Single Particle Tracking

Consider the thermal driven motion of a sphere in a complex fluid

Langevin Equation

$$m\dot{v}(t) = f(t) + \int_0^t \xi(t-t')v(t')dt'$$

Inertial force

Random thermal force

Memory function—
Material viscosity
Particle shape

Langevin Equation in Frequency Domain

Laplace transform of Langevin Equation

$$\tilde{v}(s) = \frac{\tilde{f}(s) + mv(0)}{\tilde{\xi}(s) + ms}$$

Multiple by v(0),
taking a time average,
Ignoring inertial term

$$\tilde{G}(s) = \frac{kT}{\pi as < \Delta\tilde{r}^2(s) >}$$

Random force

$$< \tilde{f}(s)v(0) >= 0$$

Equipartition of energy

$$m < v(0)v(0) >= kT$$

Generalized Stokes Einstein

$$\xi(s) = 6\pi a\tilde{\eta}(s) \quad \tilde{G}(s) = s\tilde{\eta}(s)$$

Definition and Laplace transform
of mean square displacement

$$< v(0)\tilde{v}(s) >= s^2 < \Delta\tilde{r}^2(s) > /6$$

(2) Fluorescence Laser Tracking Microrheometer

- Approach: Monitoring the Brownian dynamics of particles embedded in a viscoelastic material to probe its frequency-dependent rheology

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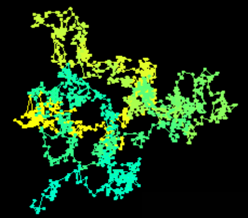


trajectory

mean squared displacement

$$\left\langle \Delta R^2(\tau) \right\rangle = \left\langle \left(\vec{r}(t+\tau) - \vec{r}(t)\right)^2 \right\rangle$$

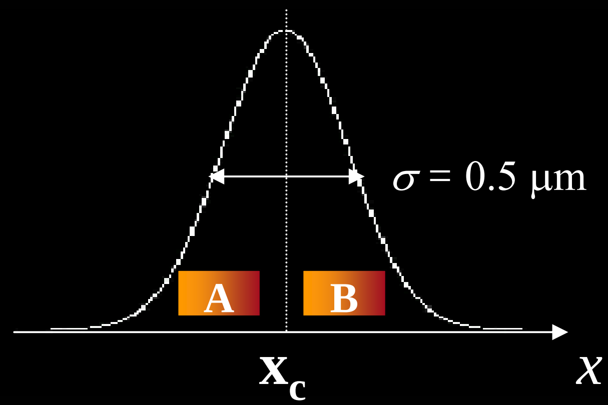
shear modulus

$$G^*(i\omega) = \frac{2k_B T}{3\pi \cdot a \cdot i\omega \cdot \left\langle \Delta \tilde{R}^2(i\omega) \right\rangle}$$

Yamada, Wirtz, Kuo, *Biophys. J.* 2000

(2) Nanometer Resolution for the Bead's Trajectory

- Collecting enough light from a fluorescent bead is critical



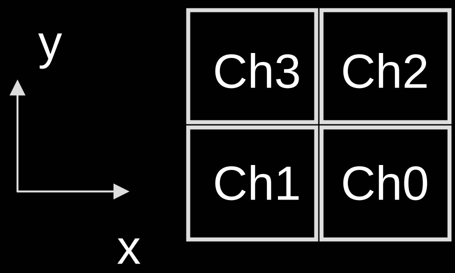
- $$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-x_c)^2}{2\sigma^2}\right]$$

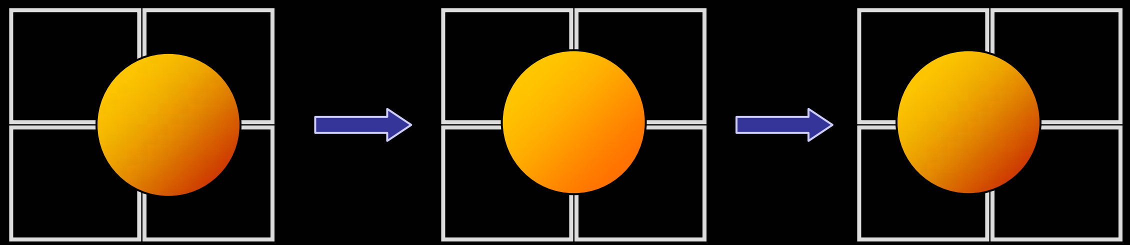
- $$\frac{N_A}{N_B} = \frac{p(A)}{p(B)} = \frac{\int_{-\infty}^{x_c} \exp\left(\frac{-x^2}{2\sigma^2}\right)dx}{\int_{x_c}^{+\infty} \exp\left(\frac{-x^2}{2\sigma^2}\right)dx}$$

Photons detected per measurement	10^3	10^4	10^5	10^6
Uncertainty on $\frac{N_A}{N_B}$	0.033	0.010	0.003	0.001
Uncertainty on x_c (nm)	12	4	1.2	0.4

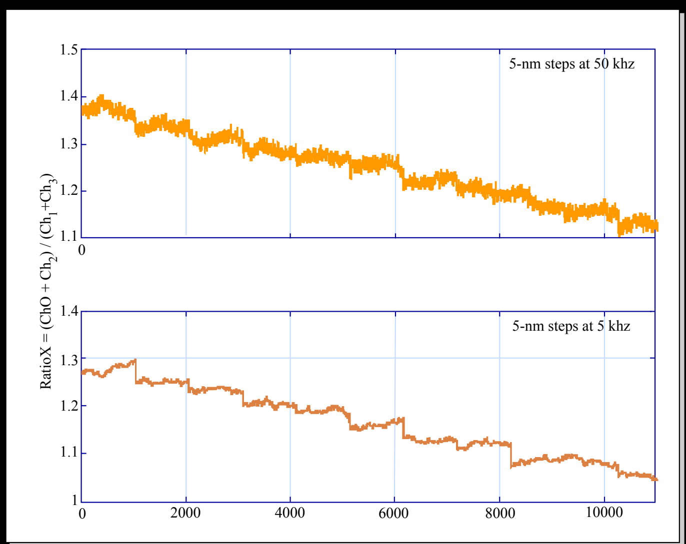
Nanometer resolution $\leftrightarrow$ 10^4 photons per measurement

(2) Calibrating the FLTM





- 5-nm stepping at 5 or 50 kHz
- Curve fitting matches theory



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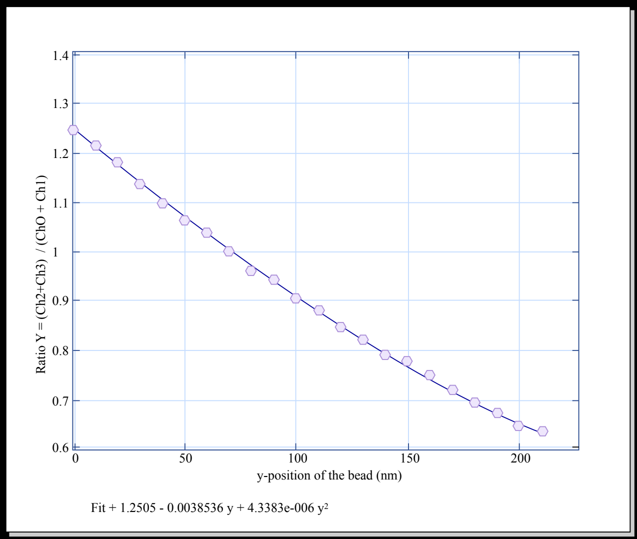

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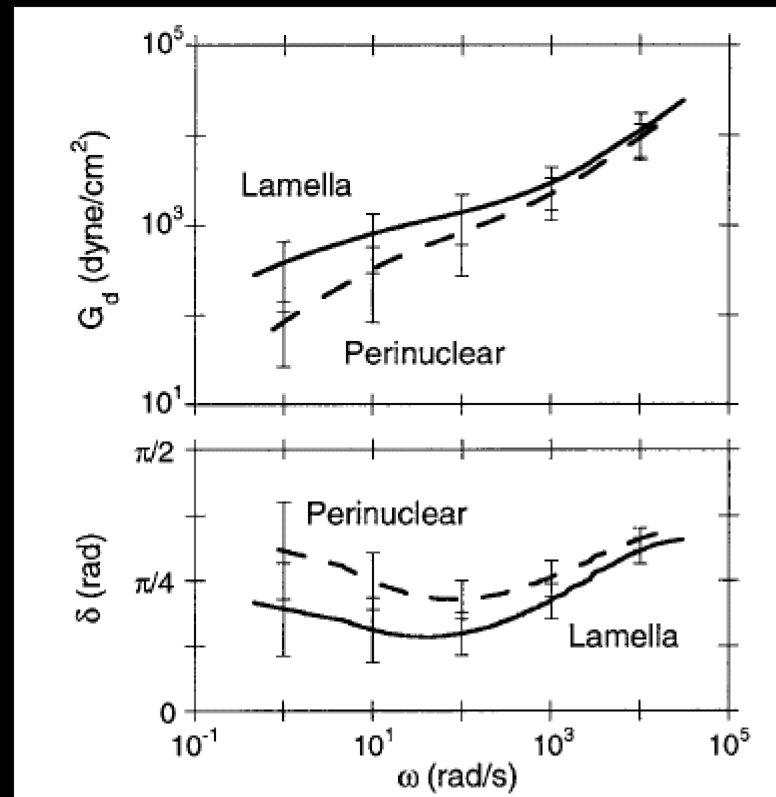
Characterizing the FLTM

- Using polyacrylamide gels (w/v 2% to 5%) of known properties

✓ Good agreement with previously published data

Single Particle Tracking Data



Two- and Multiple Particle Tracking

SPT responses can be influence by local processes (adhesion, active, etc) and not represents global cytoskeleton behavior

Solution: Look at the correlated motion of two particles under thermal force

$$D_{rr}(r,\tau) = \left\langle \Delta r_r^i(t,\tau)\,\Delta r_r^j(t,\tau)\,\delta(r - R^{ij}(t)) \right\rangle_{i\neq j,t}$$

$$D_{rr}(r,s) = \frac{kT}{2\pi r s \widetilde{G}(s)}$$

The major difference is that the correlation signal is a function of “r” the separation of the particles but not their size

Instead of using fast quadrant detectors, multiple particle tracking uses a wide field camera which is slower

SPT vs MPT

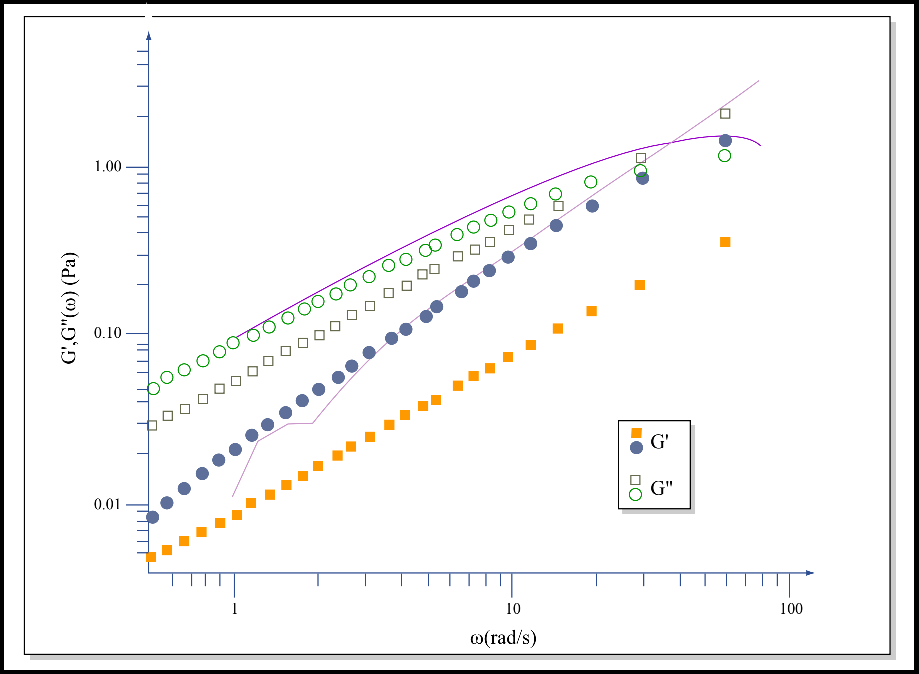


Figure by MIT OCW. After Crocker, 2000.

Triangle: SPT

Circle: MPT

SPT and MPT results can be quite different specially in cells

A Comparison of Microrheometry Methods

	Magnetic	SPT	MPT
Bandwidth	kHz	MHz	kHz
Signal Amplitude	μm	nm	nm
Local Effects	Yes	Yes	No
Nonlinear regime	Yes	No	No
Instrument	Intermediate	Intermediate	Simple